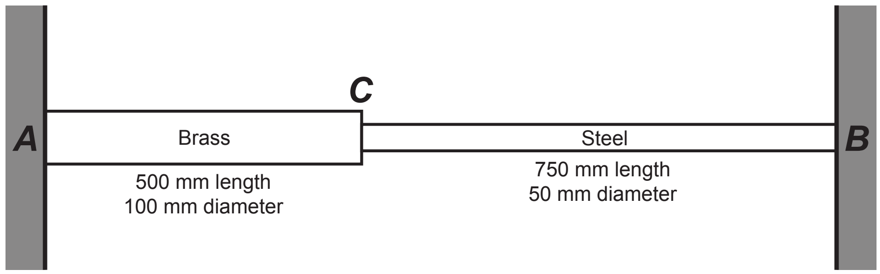
(Cheng, 10-25) A composite steel and brass bar is attached to unyielding supports at A and B with no initial stress at $25°C$, as illustrated. Find the stress in each segment when the temperature is reduced to $-5°C$ and find the length of AB, CB at $-5°C$.

	Brass	Steel
Thermal Coefficient	$\alpha = 19 \times 10^{-6}$mm/mm/$°C$	$\alpha = 12 \times 10^{-6}$mm/mm/$°C$
Young's Modulus	$E = 100$ GPa	$E = 210$ GPa



Solution:

If unrestrained (no thermal stress) and the temperature reduced to $-5°C$, there will be thermal deformation in both the brass and the steel segments. First, calculate this deformation:

$$\begin{aligned}\delta_{T_{br}} &= \alpha \cdot L \cdot \Delta T \\ &= 19 \times 10^{-6} \times 500 \times (-30) \\ &= -0.285 \text{ mm} \\ \delta_{T_{st}} &= 12 \times 10^{-6} \times 500 \times (-30) \\ &= -0.270 \text{ mm}\end{aligned}$$

Therefore, the unrestrained lengths of the segments at $-5°C$ are:

$$\begin{aligned}L_{br} &= 500 - 0.285 \\ &= 499.715 \text{ mm} \\ L_{st} &= 750 - 0.270 \\ &= 749.73 \text{ mm}\end{aligned}$$

But the rod is not unrestrained. Unyielding supports at A and B apply a tensile load that maintains the rod length at $500 + 750 = 1250$ mm. This tensile load, P, causes both AC and CB to deform but we don't know how much each segment deforms. This is a statically indeterminate problem.

We have $\Sigma F_x = P_B - P_A = 0$, so we can write $P_A = P_B = P$. This is a statically indeterminate problem.

We do know that the sum of the deformations is $0.285 + 0.270 = 0.555$ mm. (Note that this is no longer thermal deformation; it is deformation due to the applied tensile force.)

$$\begin{aligned}\delta_{br} + \delta_{st} &= 0.555 \\ 0.555 &= \left(\frac{PL}{AE}\right)_{br} + \left(\frac{PL}{AE}\right)_{st} \\ &= \frac{P \times 499.715}{\frac{\pi(100)^2}{4} \times 100 \times 10^3} + \frac{P \times 749.73}{\frac{\pi(50)^2}{4} \times 210 \times 10^3} \\ P &= 226.11 \text{ kN}\end{aligned}$$

Now we can find the stresses in AC and in CB:

$$\begin{aligned}
\sigma_{AC} &= \frac{226110}{\frac{\pi(100)^2}{4}} \\
&= 28.789 \text{ MPa} \\
\sigma_{CB} &= \frac{226110}{\frac{\pi(50)^2}{4}} \\
&= 115.157 \text{ MPa}
\end{aligned}$$

What is the length of AC and CB at $-5°C$?

$$\begin{aligned}
\delta_{AC} &= \frac{PL}{AE} \\
&= \sigma_{AC} \cdot \frac{L}{E} \\
&= 28.789 \times \frac{499.715}{100 \times 10^3} \\
&= 0.14387 \text{ mm} \\
\delta_{CB} &= \sigma_{CB} \cdot \frac{749.73}{210 \times 10^3} \\
&= 0.41113 \text{ mm.} \\
L_{AC} &= 499.715 + 0.14386 \\
&= 499.85886 \text{ mm} \\
L_{CB} &= 749.73 + 0.41113 \\
&= 750.14113
\end{aligned}$$

Then the total length is $499.85886 + 750.141113 = 1249.9999$. Note that although restrained, with the same overall length, the junction between the brass and steel has shifted 0.14114 mm to the left!