Method of Diagrams

(Strength of Materials)

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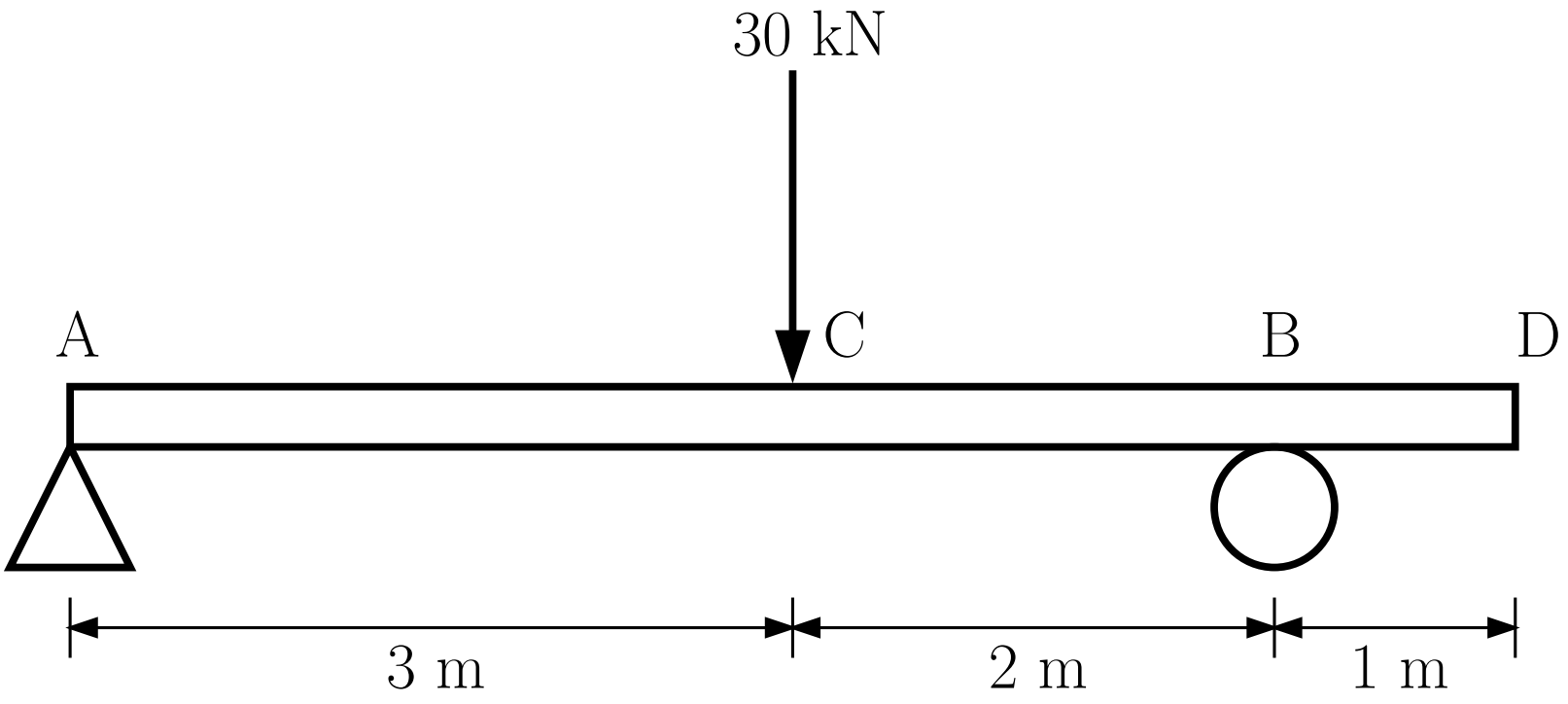
- We have used the Method of Sections to calculate the shear force, V, and the bending moment, M', at various locations along the length of a beam

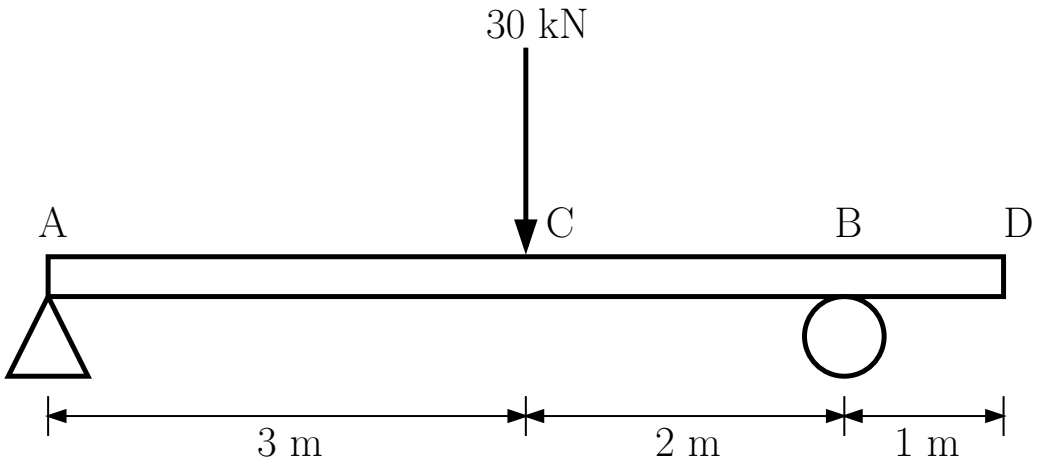
- Frequently, we need to know the shear forces and the bending moments at *every* point along the beam

- We can graph the the shear forces and bending moments using the Method of Diagrams

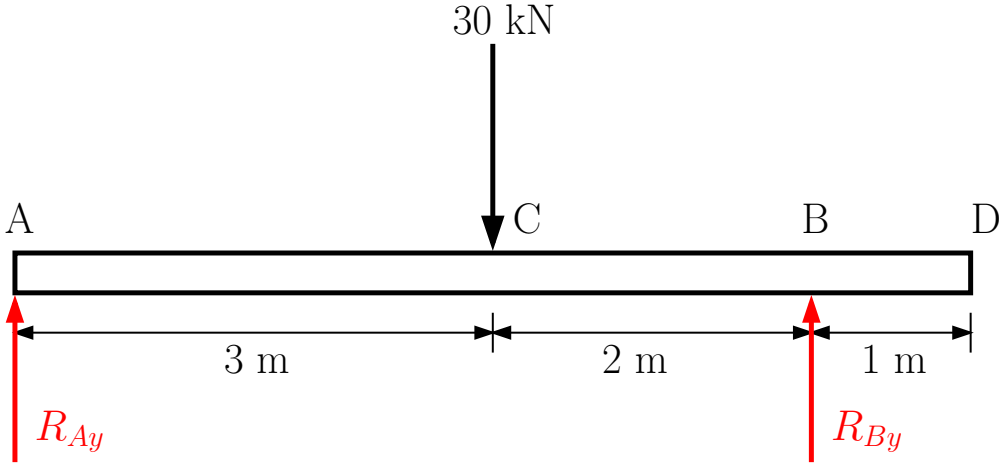
Example:

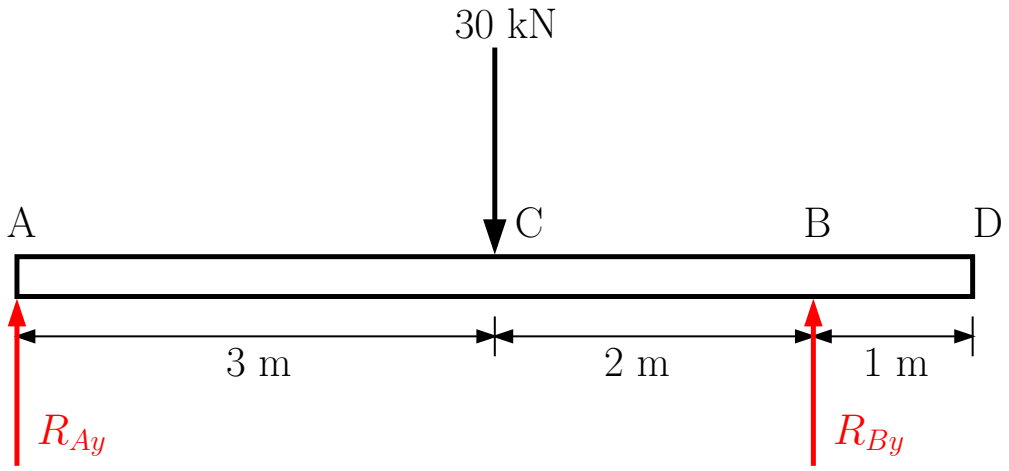


Consider a simply-supported beam, with a concentrated load of 30 kN, as illustrated.



Start by drawing the FBD:





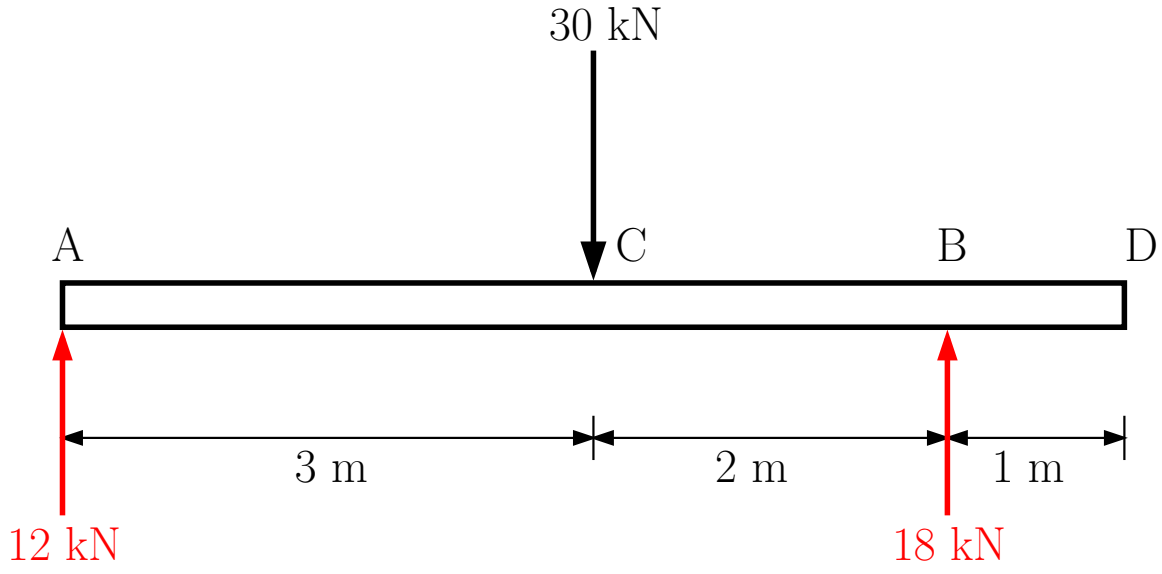
Then solve for the reaction forces at A and at B:

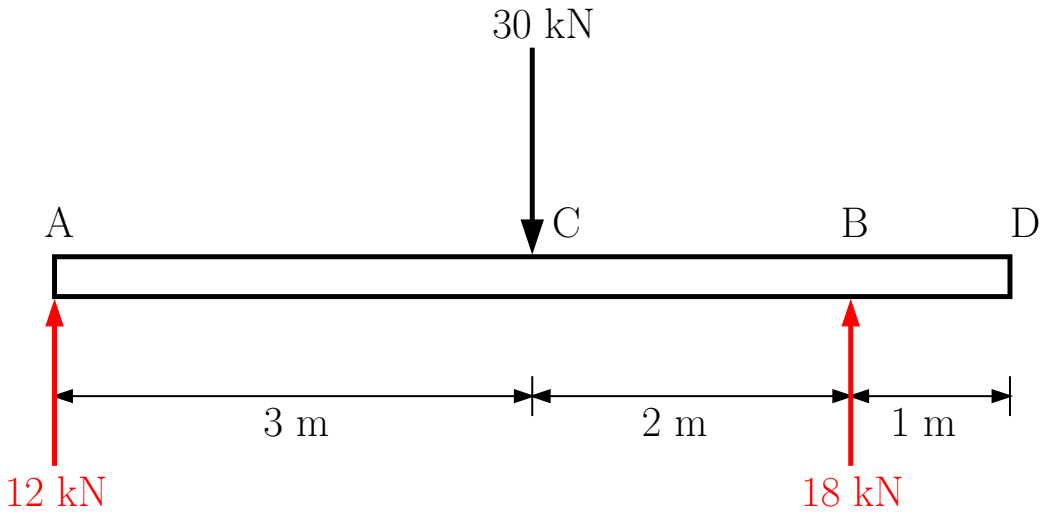
$$\begin{aligned} \Sigma M_A &= R_{By} \times 5 - 30 \times 3 &&= 0 \\ R_{By} &= &&= 18 \\ \Sigma M_B &= 30 \times 2 - R_{Ay} \times 5 &&= 0 \\ R_{Ay} &= &&= 12 \end{aligned}$$

We verify our results:

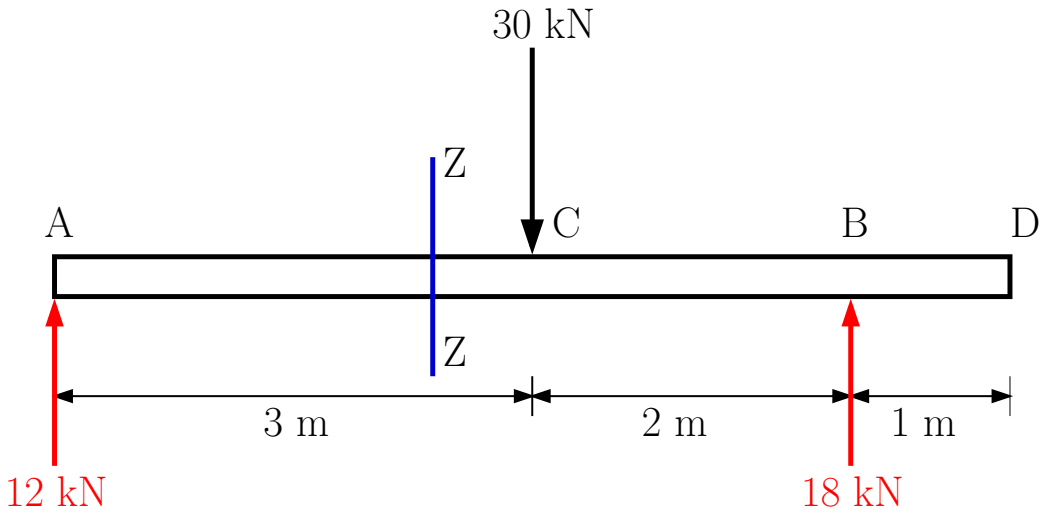
$$\begin{aligned} \Sigma F_y &= R_{Ay} + R_{By} - 30 \\ &= 12 + 18 - 30 \\ &= 0 \end{aligned}$$

Thus,



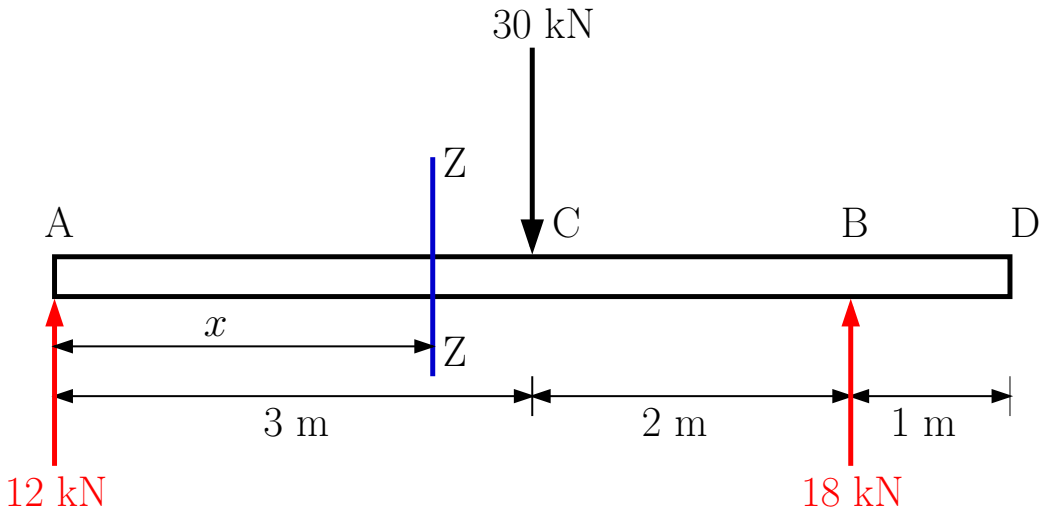


First, look at the segment from A to C:



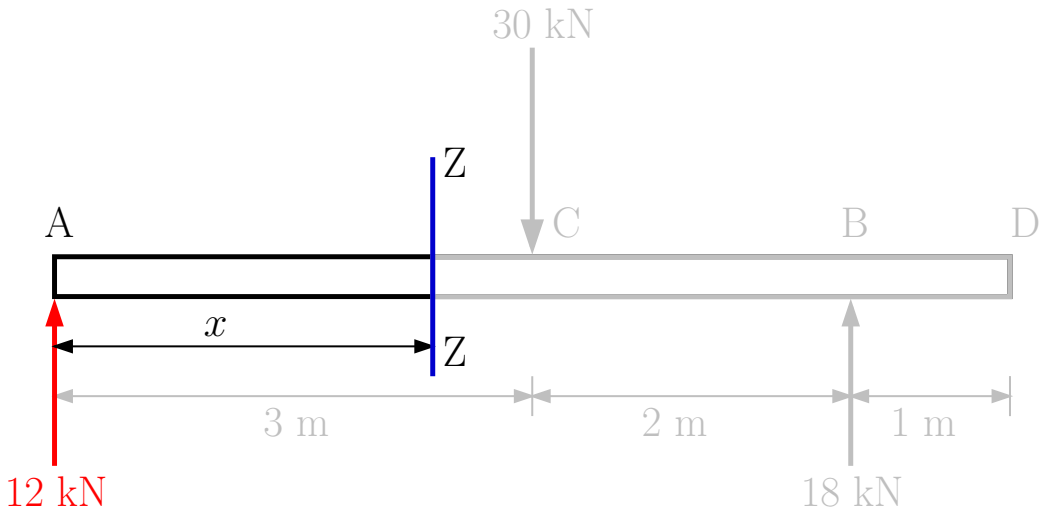
First, look at the segment from A to C:

- Draw a section ZZ through the beam between A and C



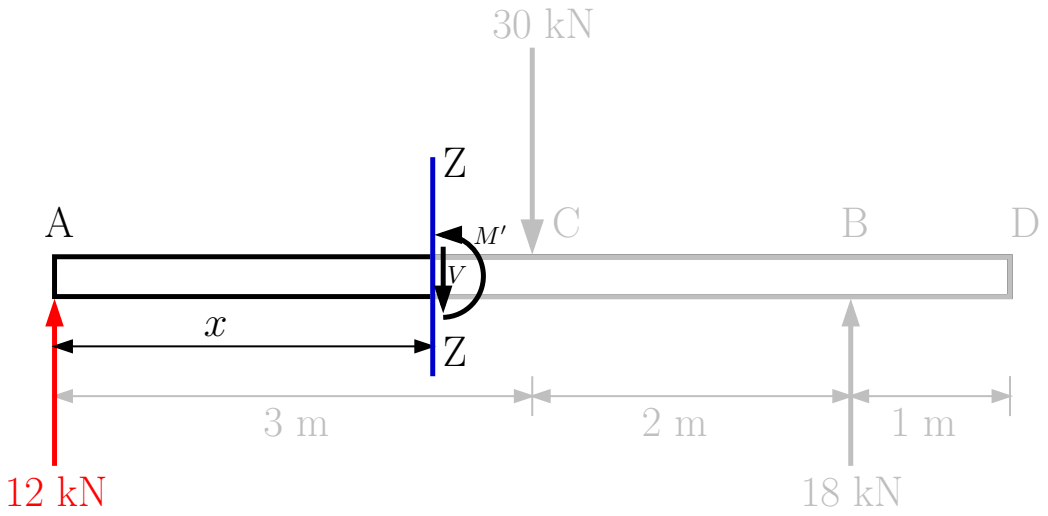
First, look at the segment from A to C:

- Draw a section ZZ through the beam between A and C
- Let ZZ be a distance x from A; then $0 < x < 3$



First, look at the segment from A to C:

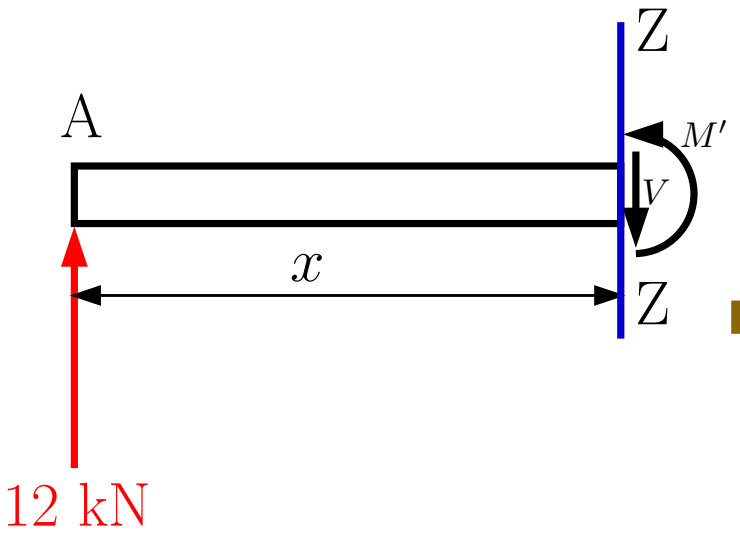
- Draw a section ZZ through the beam between A and C
- Let ZZ be a distance x from A; then $0 < x < 3$
- Disregard the beam to the right of ZZ



First, look at the segment from A to C:

- Draw a section ZZ through the beam between A and C
- Let ZZ be a distance x from A; then $0 < x < 3$
- Disregard the beam to the right of ZZ
- Consider the shear force V and the bending moment at ZZ to be positive

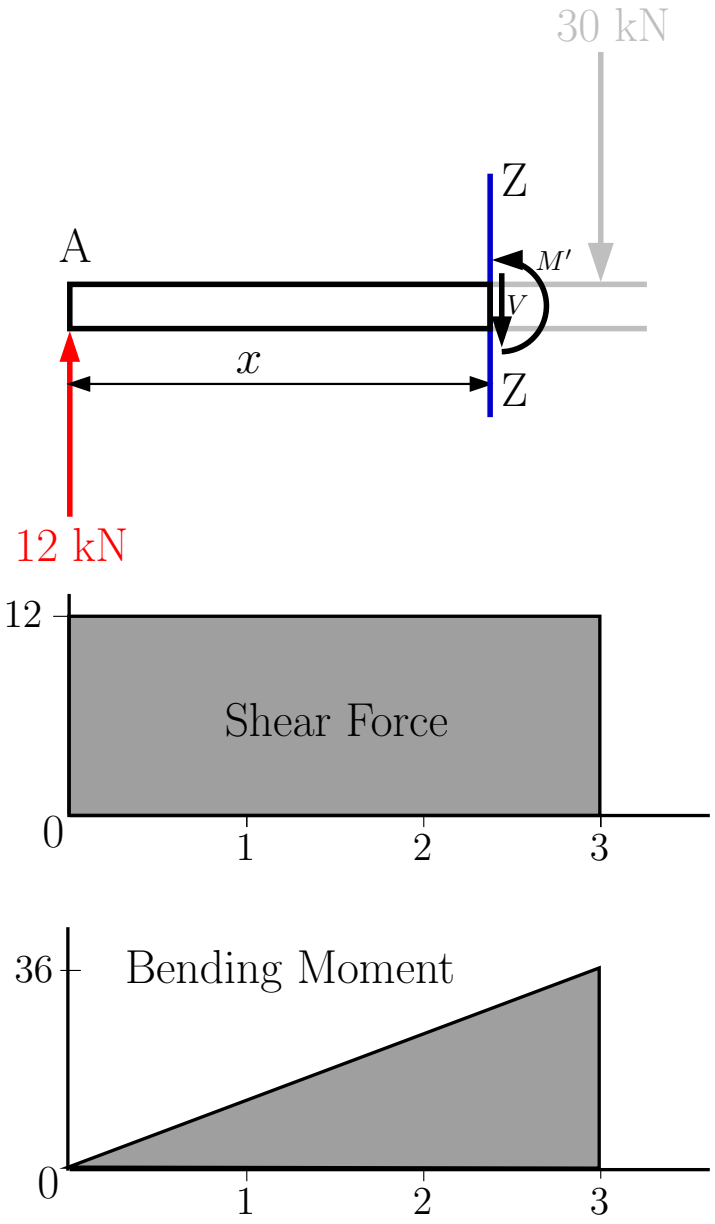
- To find the shear force V, sum the y components of forces between A and ZZ



$$\begin{aligned} \Sigma F_y &= 12 - V &&= 0 \\ V &= 12 \text{ kN} \end{aligned}$$

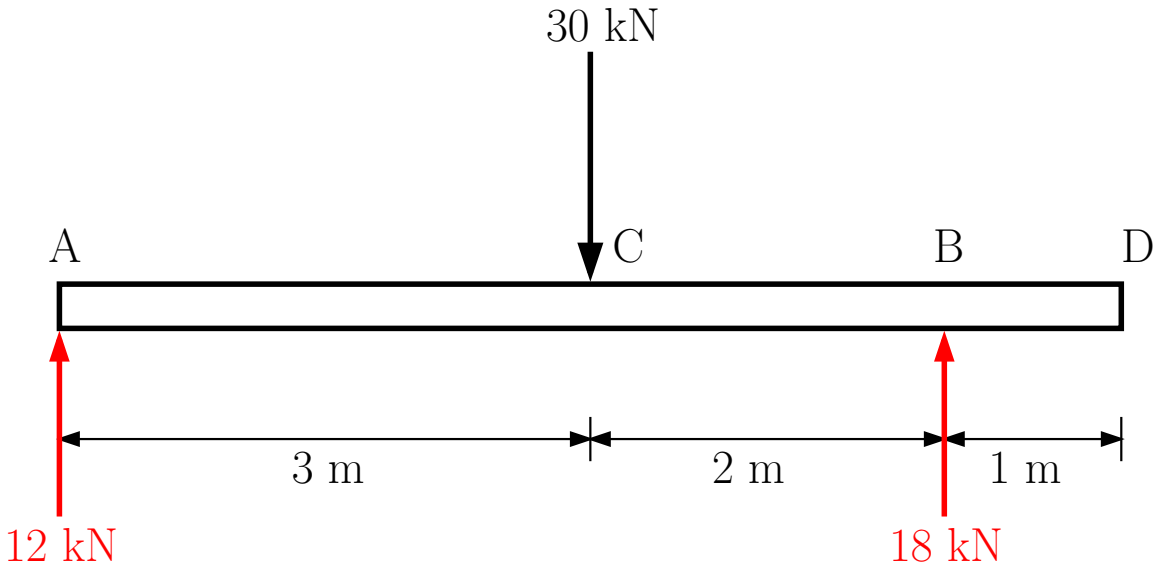
- To find the bending moments, sum the moments about the section ZZ

$$\begin{aligned} \Sigma M_{ZZ} &= M' - 12 \times x &&= 0 \\ M' &= 12x \text{ kN} \cdot \text{m} \end{aligned}$$

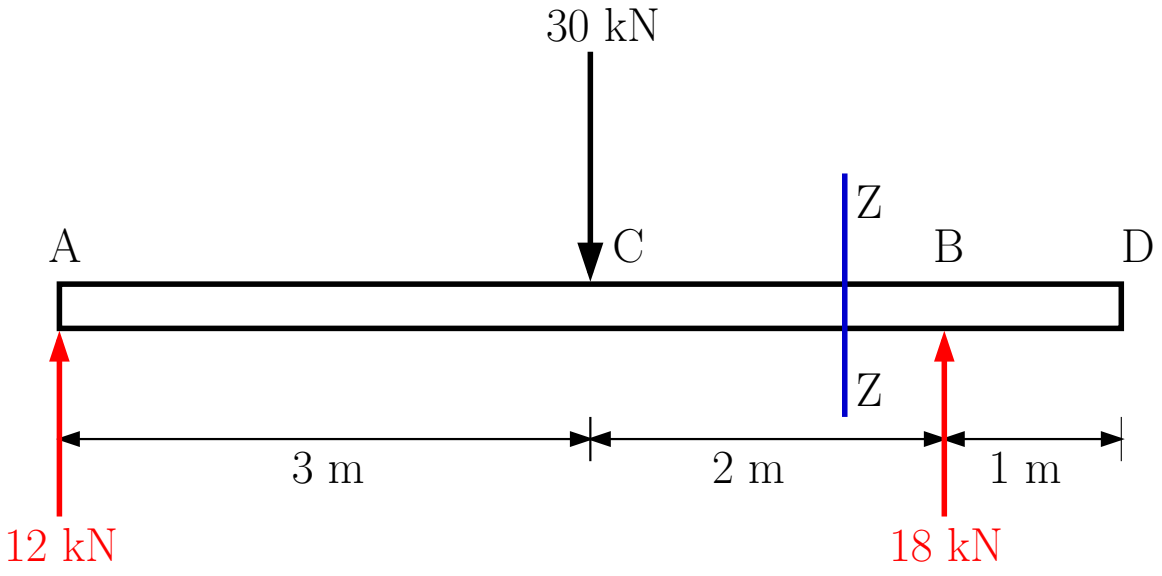


- $V = 12$ kN for all values of x such that $0 < x < 3$.
  The shear force is **constant** over this range.

- $M' = 12x$ kN $\cdot$ m so the bending moment varies **linearly** with x

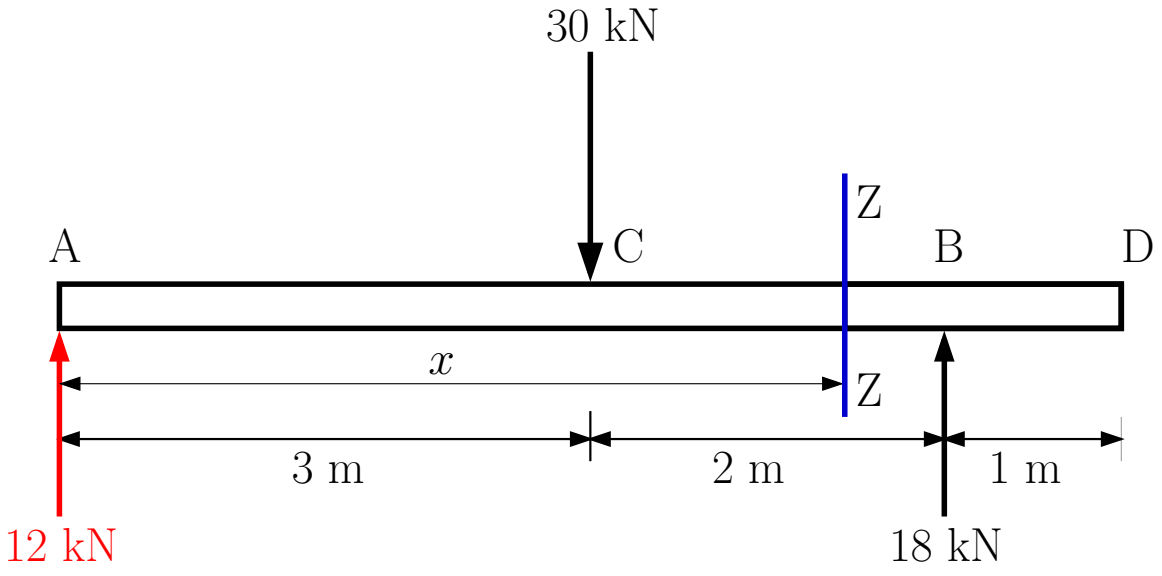


Now look at the segment from C to B:



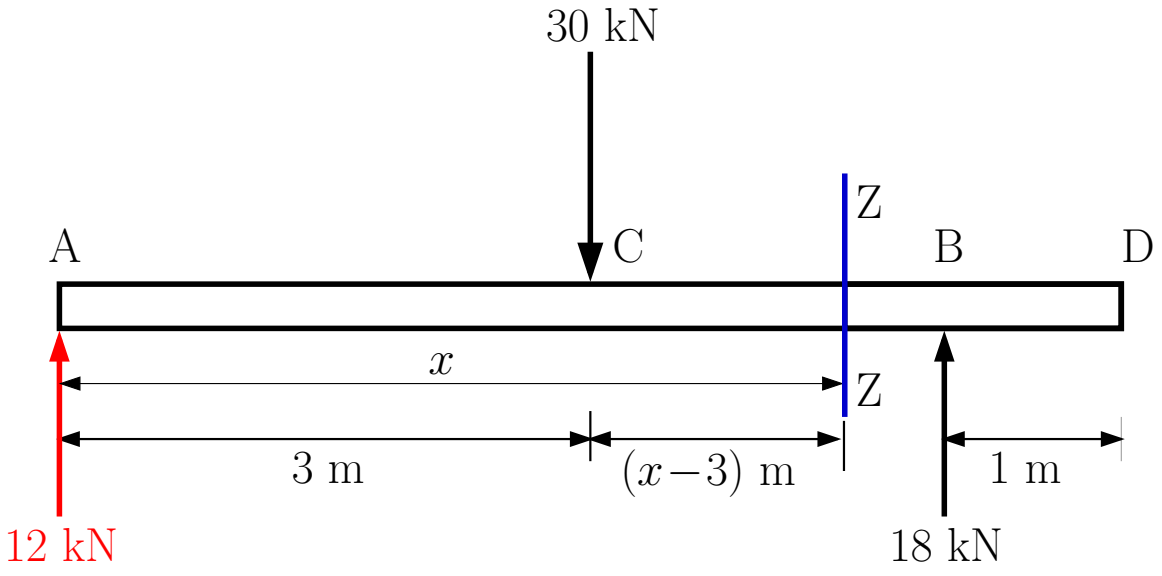
Now look at the segment from C to B:

- Draw a section ZZ through the beam between C and B



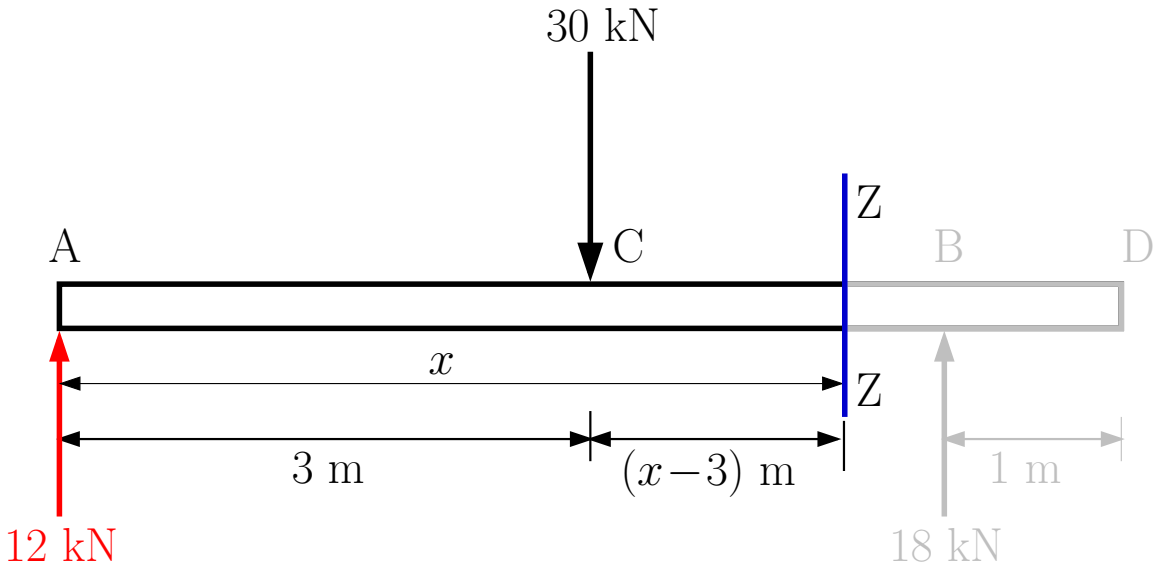
Now look at the segment from C to B:

- Draw a section ZZ through the beam between C and B
- Let ZZ be a distance x from A; then $3 < x < 5$



Now look at the segment from C to B:

- Draw a section ZZ through the beam between C and B
- Let ZZ be a distance x from A; then $3 < x < 5$
- The distance from ZZ to C is $x - 3$



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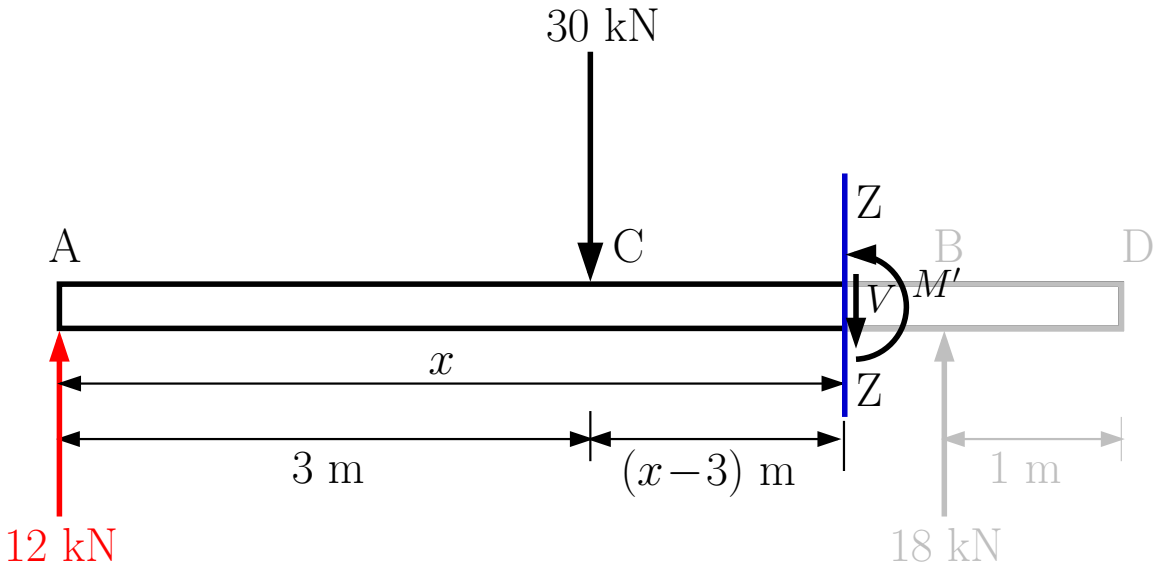
- Disregard the beam to the right of ZZ



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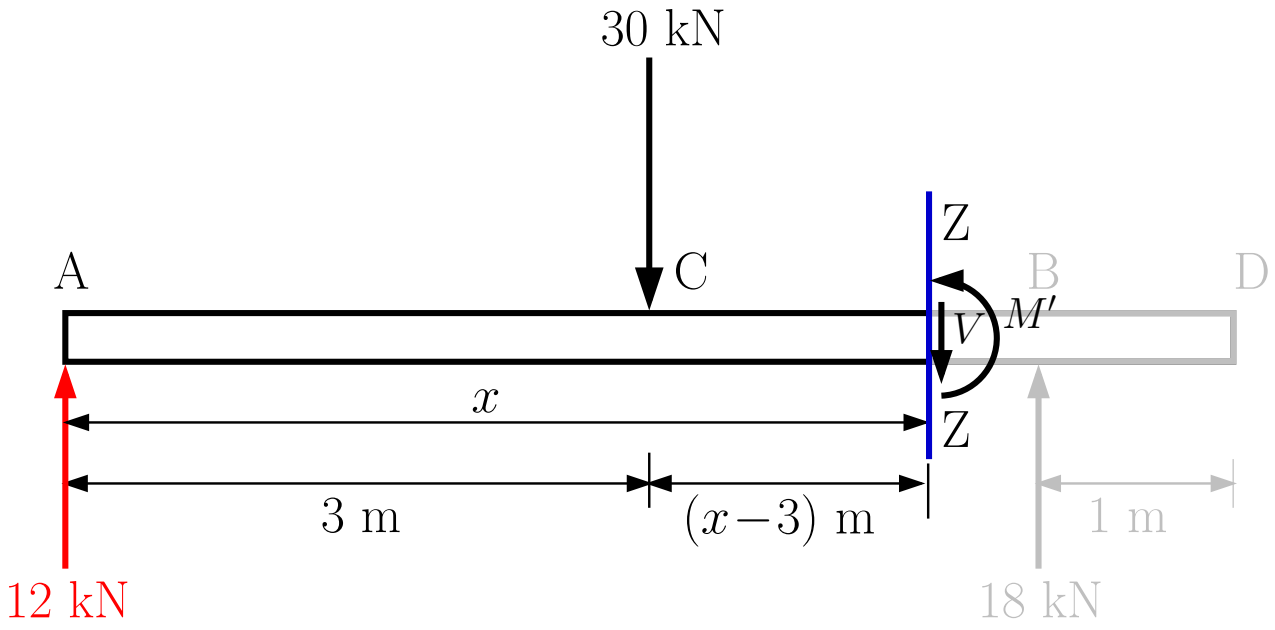
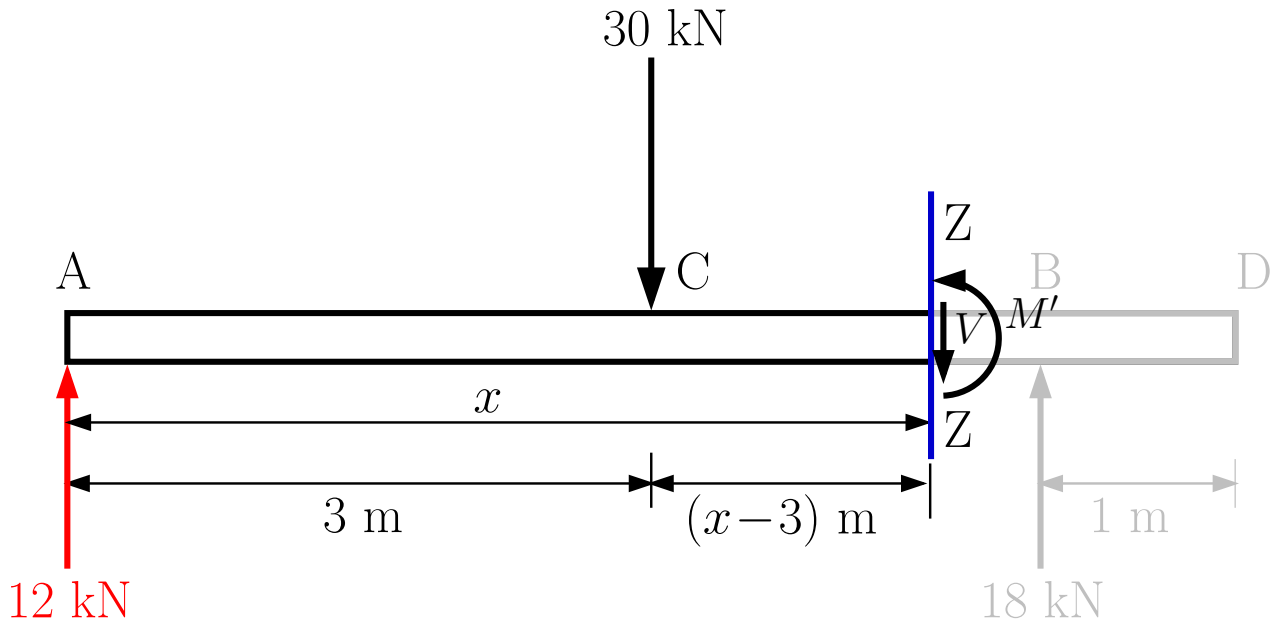


$$\Sigma F_y = 12 - 30 - V = 0$$

$$V = -18 \text{ kN}$$

We have $$V = \begin{cases} 12 & \text{if } 0 < x < 3 \\ -18 & \text{if } 3 < x < 5 \end{cases}$$

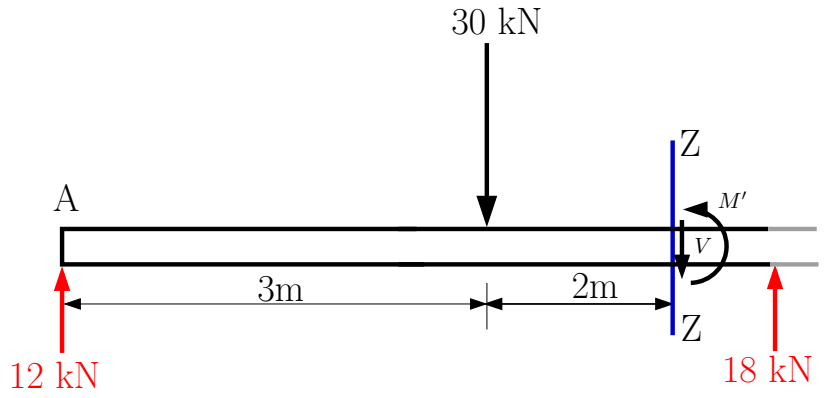


$$\Sigma M_{ZZ} = M' + 30(x-3) - 12x = 0$$

$$M' = 12x - 30(x-3)$$

$$M' = 90 - 18x \text{ kN} \cdot \text{m}$$

We have $$M' = \begin{cases} 12x & \text{if } 0 < x < 3 \\ 90 - 18x & \text{if } 3 < x < 5 \end{cases}$$



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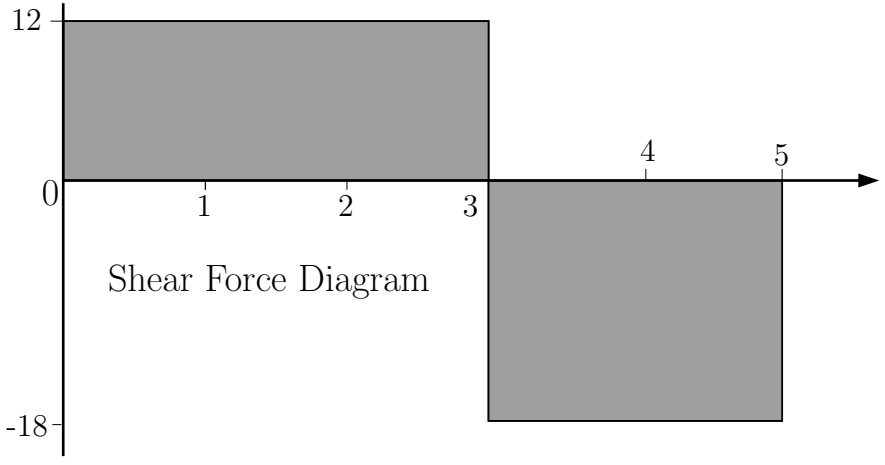

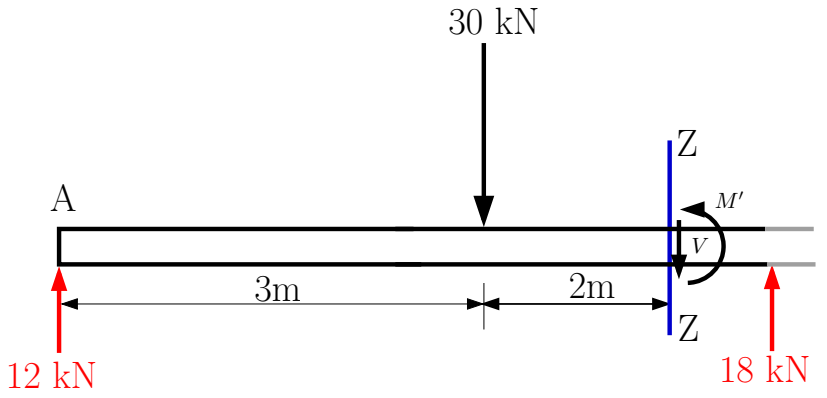

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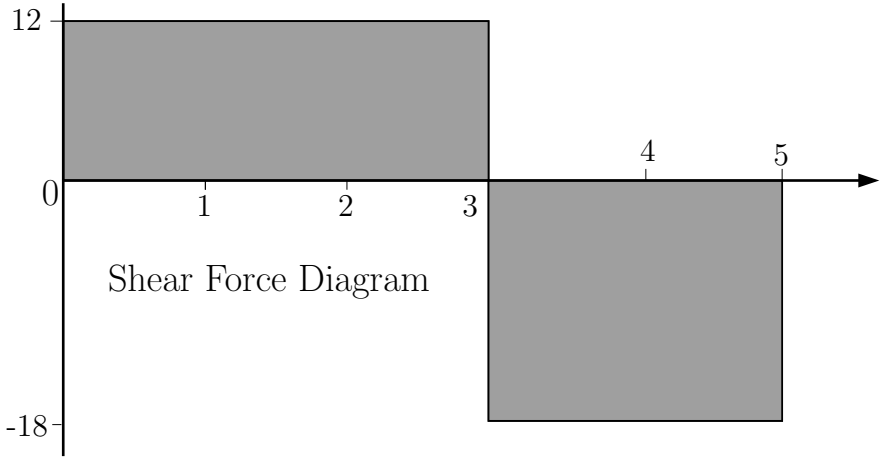
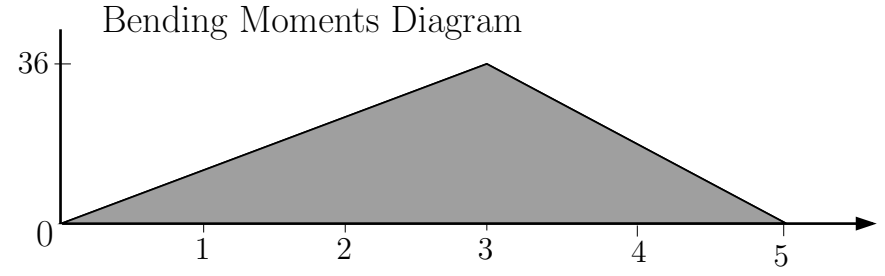
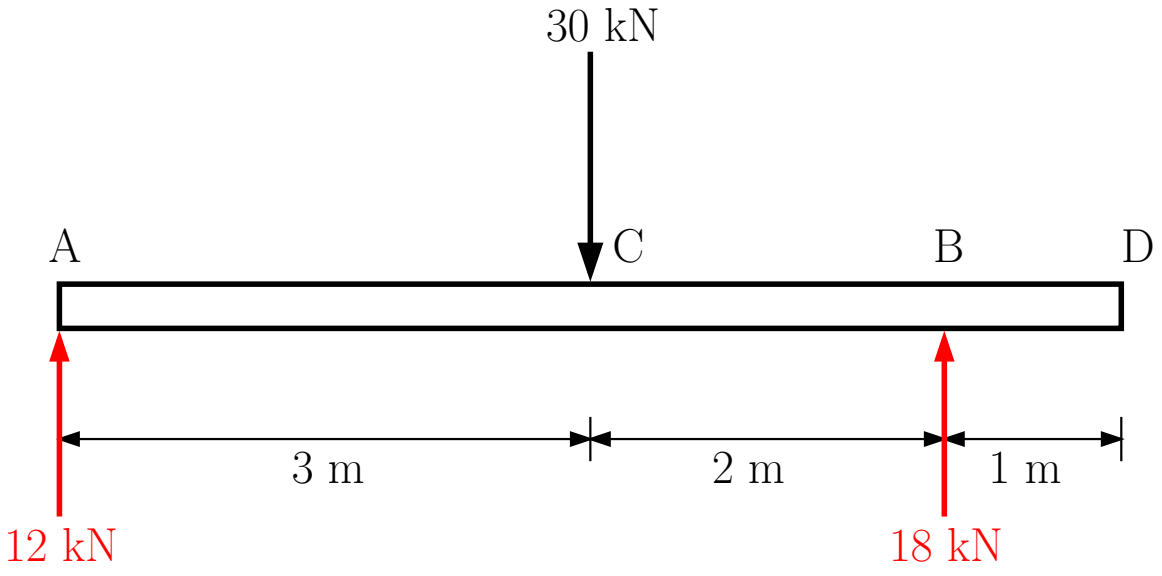
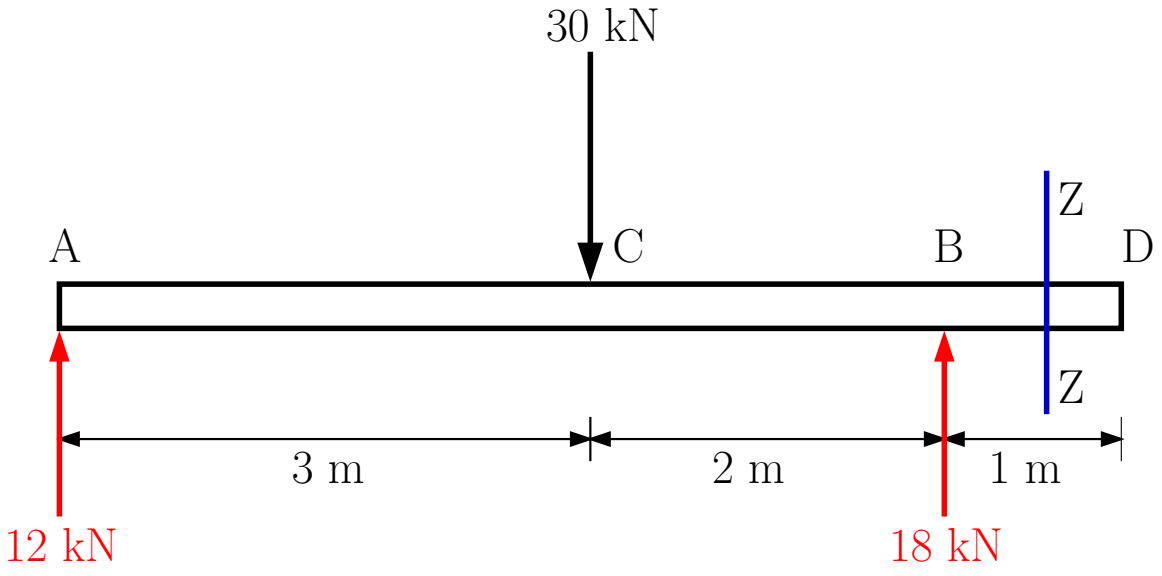


$$M' = \begin{cases} 12x & \text{if } 0 < x < 3 \\ 90 - 18x & \text{if } 3 < x < 5 \end{cases}$$



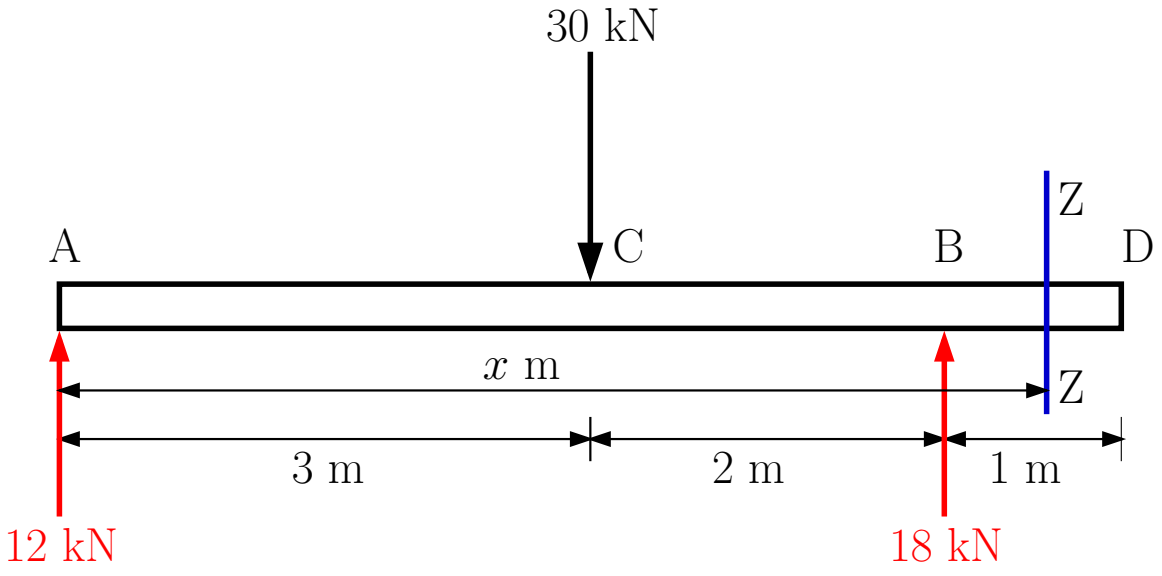


Now look at the segment from B to D:



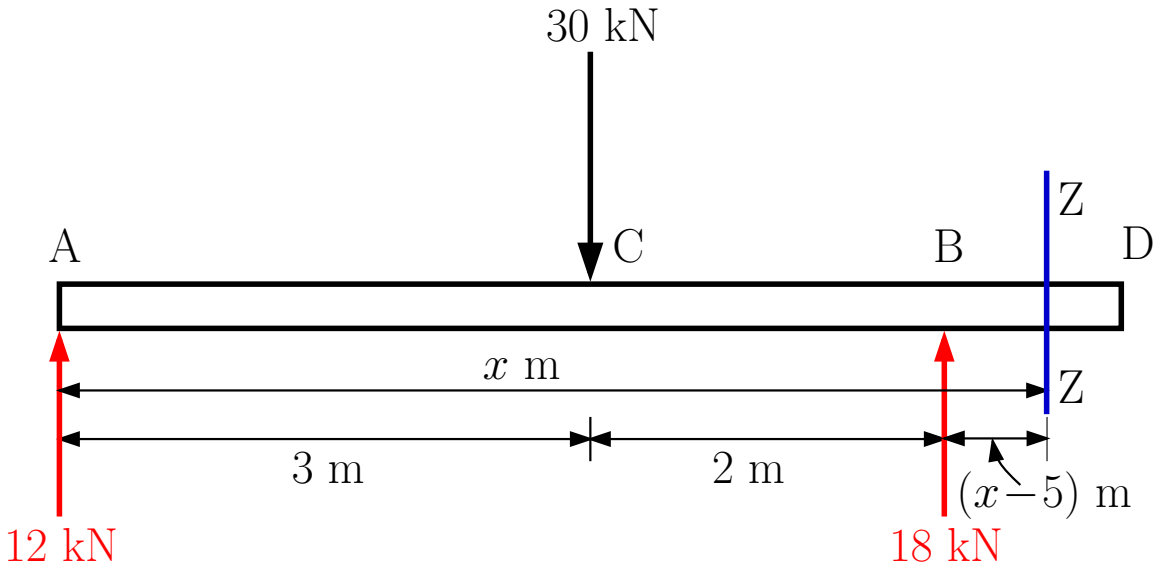
Now look at the segment from B to D:

- Draw a section ZZ through the beam between B and D



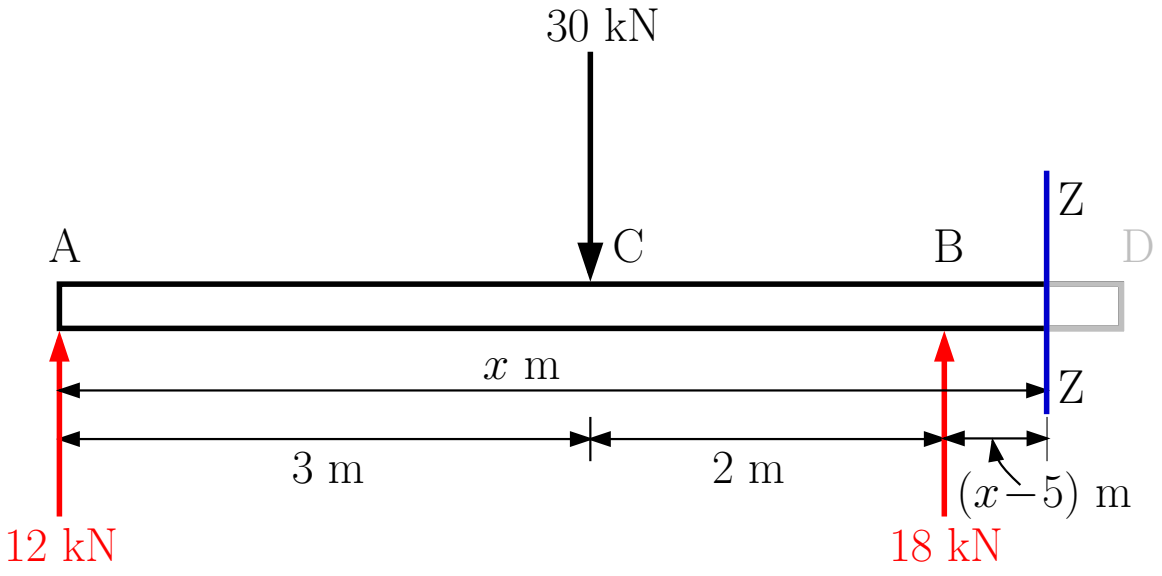
Now look at the segment from B to D:

- Draw a section ZZ through the beam between B and D
- Let ZZ be a distance x from A; then $5 < x < 6$



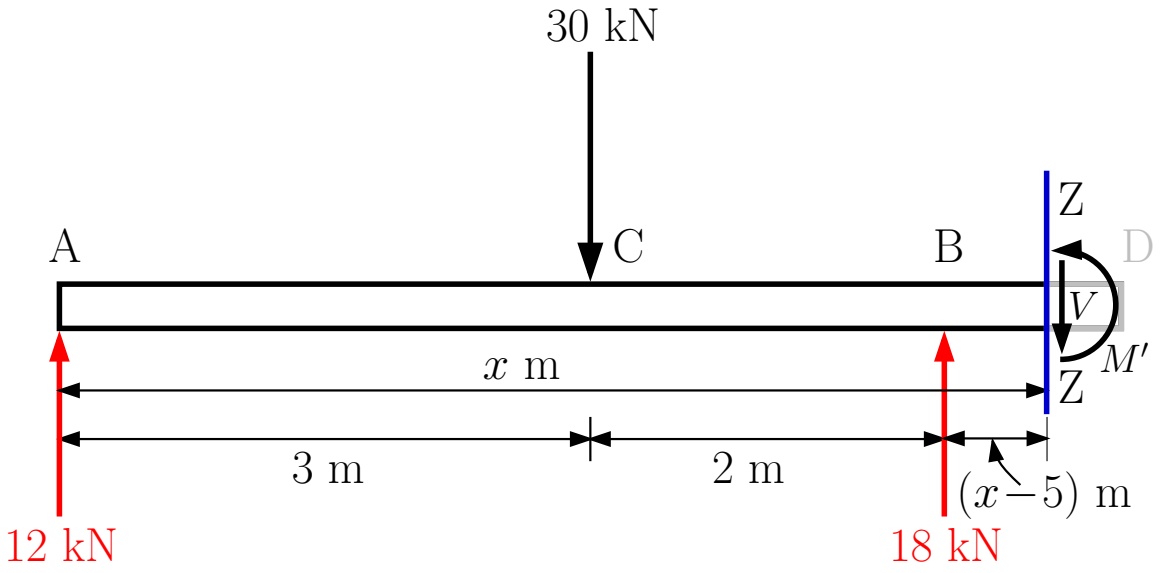
Now look at the segment from B to D:

- Draw a section ZZ through the beam between B and D
- Let ZZ be a distance x from A; then $5 < x < 6$
- The distance from ZZ to B is $x - 5$



Now look at the segment from B to D:

- Draw a section ZZ through the beam between B and D
- Let ZZ be a distance x from A; then $5 < x < 6$
- The distance from ZZ to B is $x - 5$
- Disregard the beam to the right of ZZ



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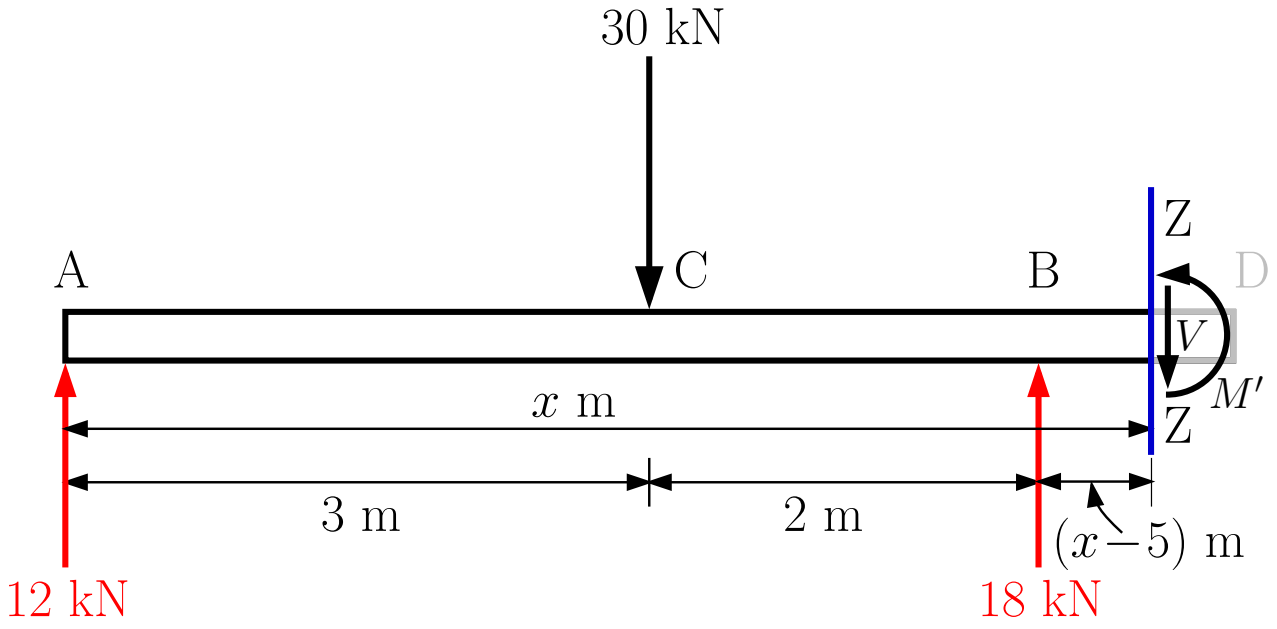
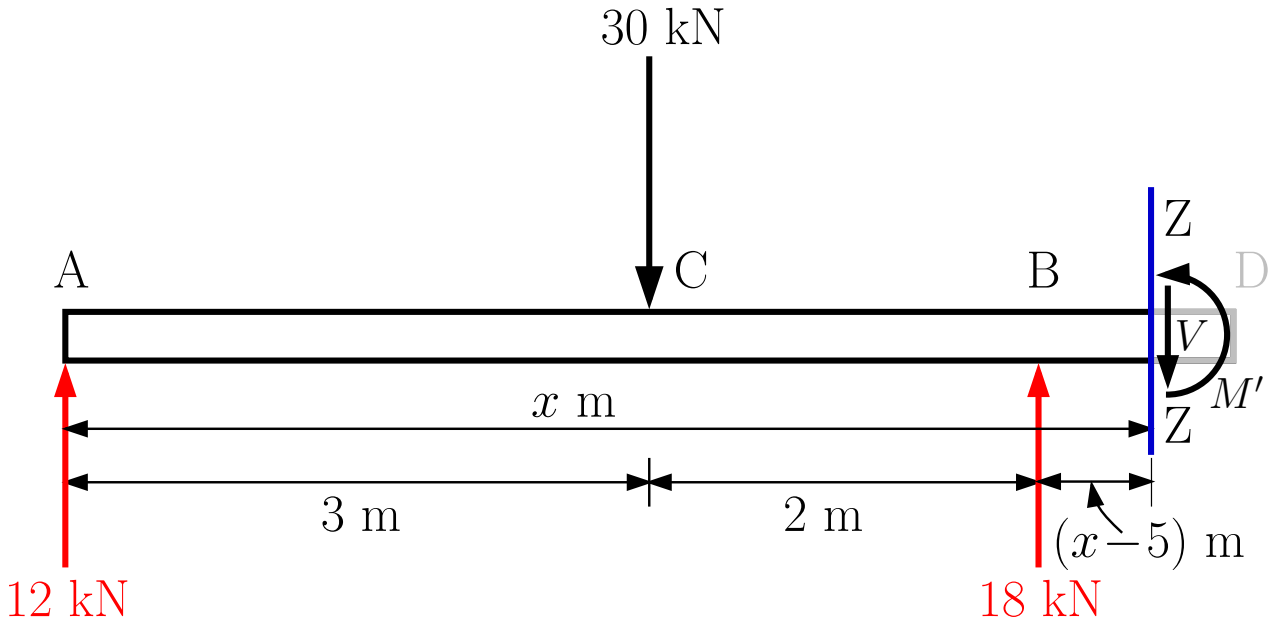
- Disregard the beam to the right of ZZ
- Consider the shear force V and the bending moment at ZZ to be positive



$$\Sigma F_y = 12 - 30 + 18 - V = 0$$

$$V = 0 \text{ kN}$$

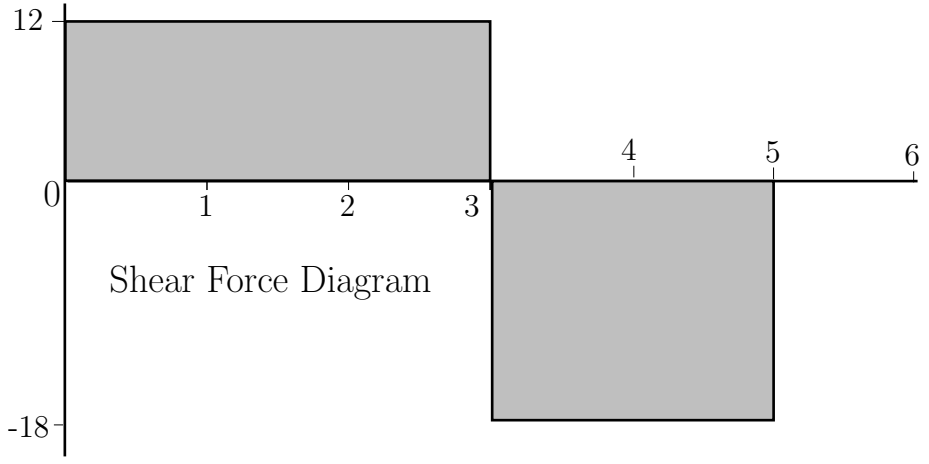
We have $$V = \begin{cases} 12 & \text{if } 0 < x < 3 \\ -18 & \text{if } 3 < x < 5 \\ 0 & \text{if } 5 < x \leq 6 \end{cases}$$



$$\begin{aligned}
\Sigma M_{ZZ} &= M' + 30(x-5+2) - 12x - 18(x-5) &&= 0 \\
\Rightarrow \quad & M' + 30x - 90 - 12x - 18x + 90 &&= 0 \\
& M' &&= 0 \text{ kN} \cdot \text{m}
\end{aligned}$$

We have $$M' = \begin{cases} 12x & \text{if } 0 < x < 3 \\ 90 - 18x & \text{if } 3 < x < 5 \\ 0 & \text{if } 5 < x \leq 6 \end{cases}$$

$$V = \begin{cases} 12 & \text{if } 0 < x < 3 \\ -18 & \text{if } 3 < x < 5 \\ 0 & \text{if } 5 < x \leq 6 \end{cases}$$



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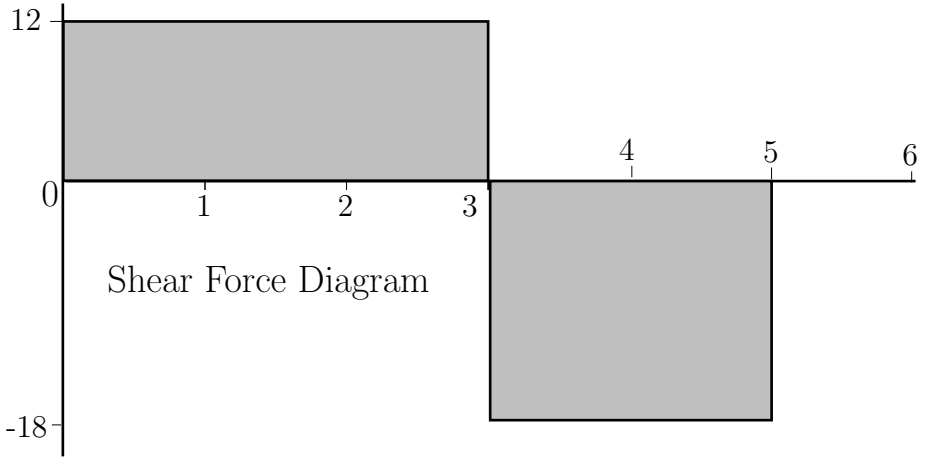

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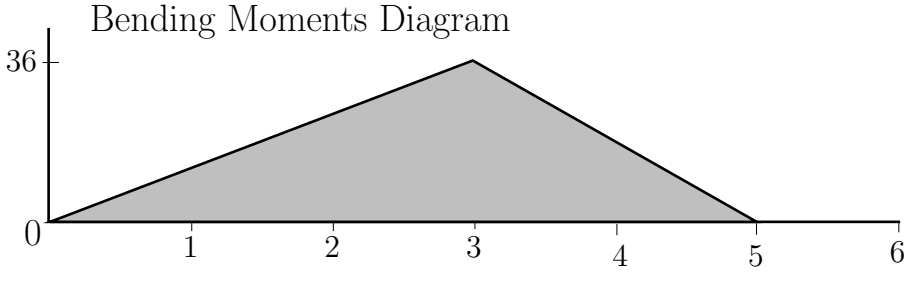

Exercise:

Created by Dave Morgan using LaTeX 2_ε and *Prosper* on March 15, 2005