
Method of Diagrams

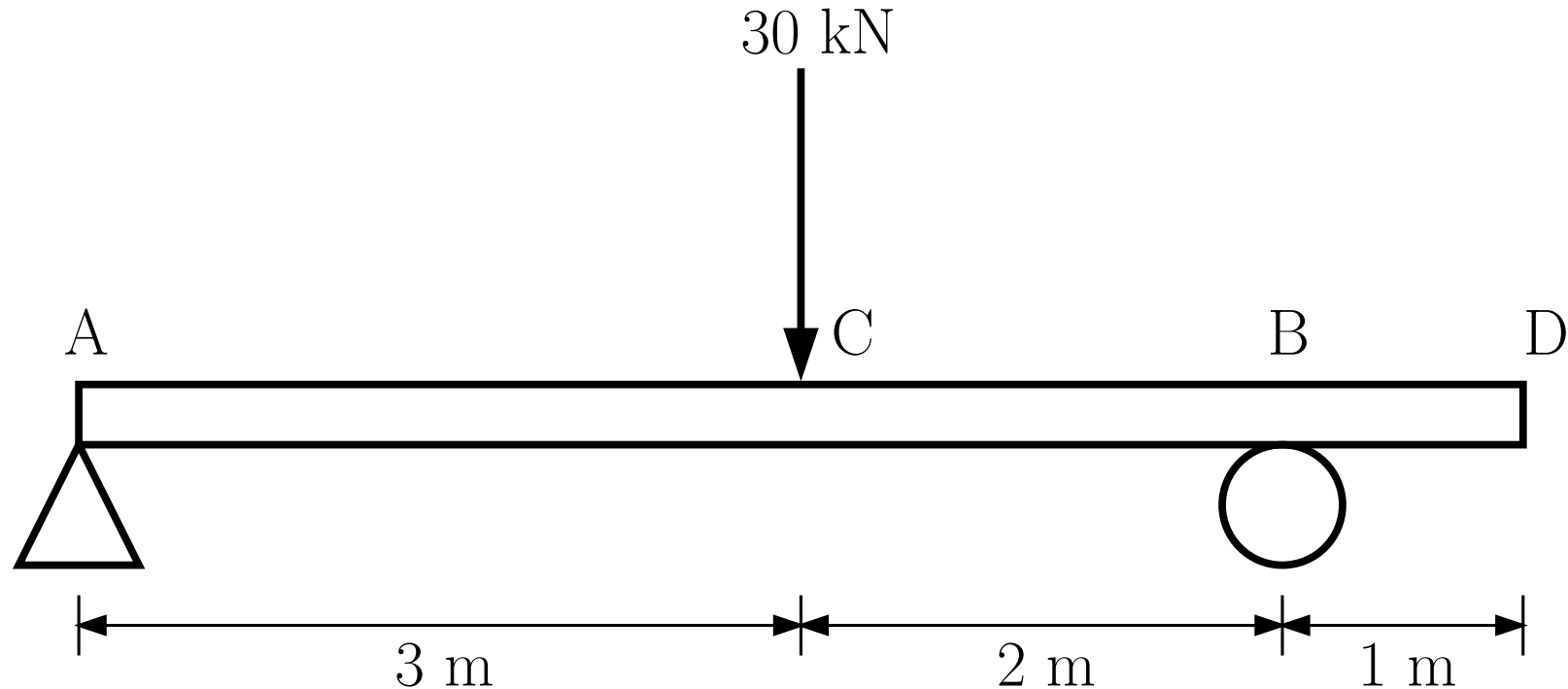
(Strength of Materials)

Dave Morgan

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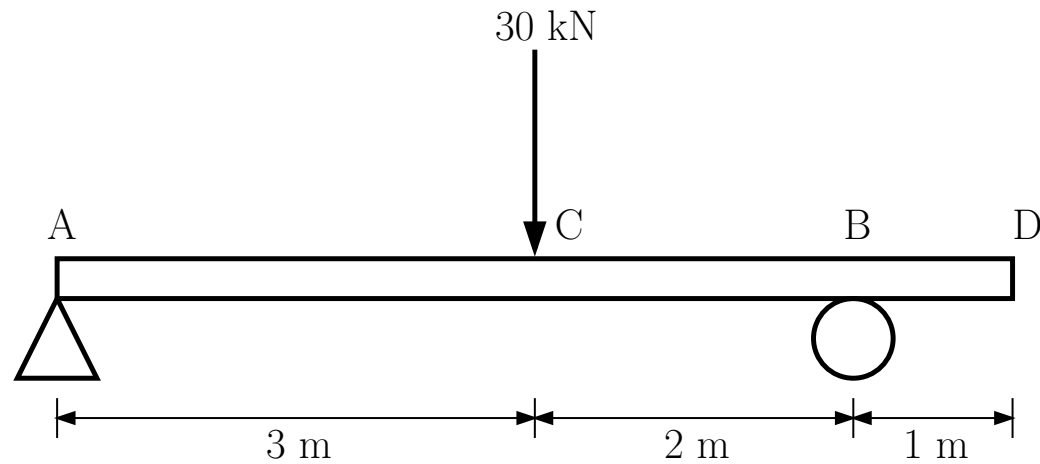
- We have used the Method of Sections to calculate the shear force, V , and the bending moment, M' , at various locations along the length of a beam
- Frequently, we need to know the shear forces and the bending moments at *every* point along the beam
- We can graph the the shear forces and bending moments using the Method of Diagrams

Example:

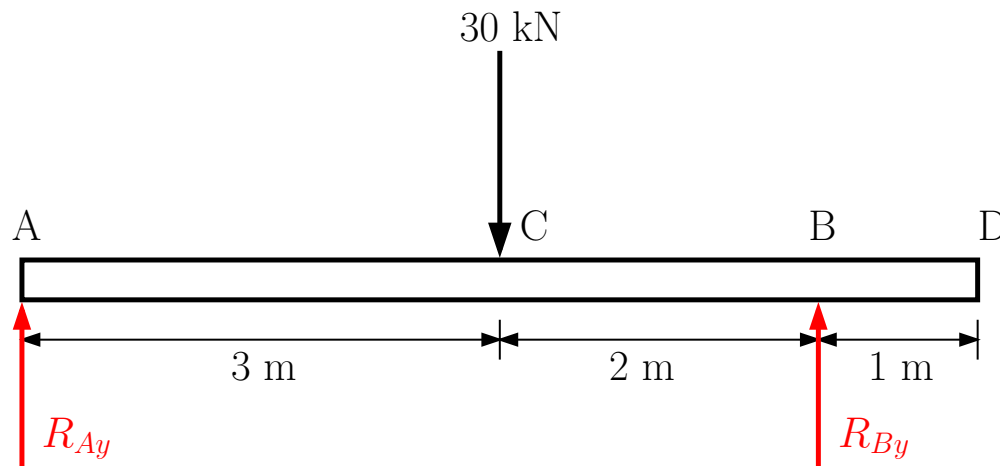


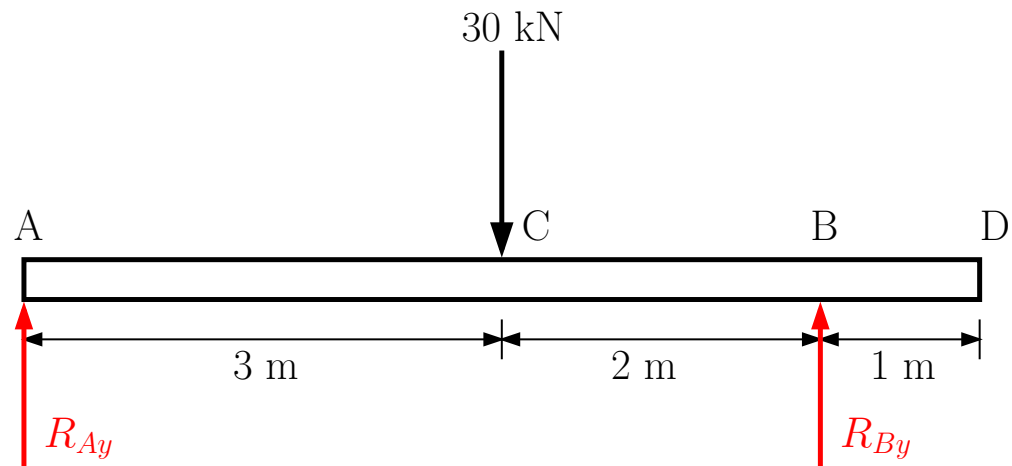
Consider a simply-supported beam, with a concentrated load of 30 kN, as illustrated.

Method of Diagrams



Start by drawing the FBD:





Then solve for the reaction forces at A and at B :

$$\Sigma M_A = R_{By} \times 5 - 30 \times 3 = 0$$

$$R_{By} = 18$$

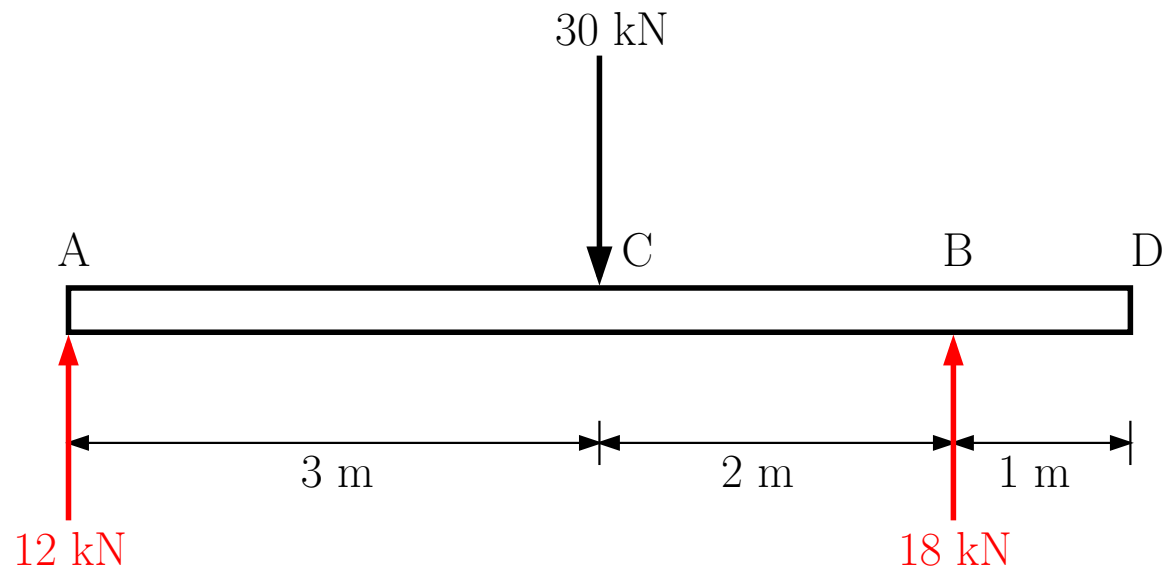
$$\Sigma M_B = 30 \times 2 - R_{Ay} \times 5 = 0$$

$$R_{Ay} = 12$$

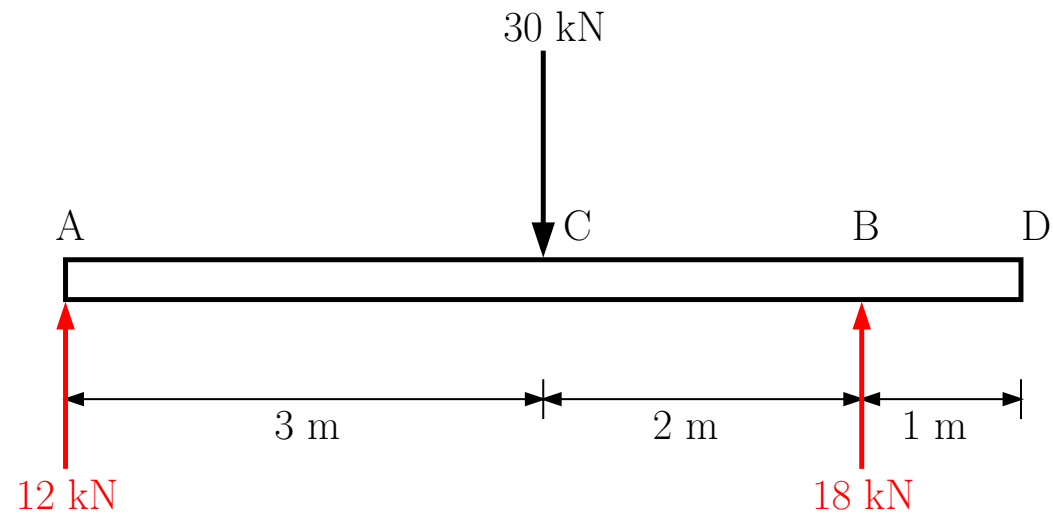
We verify our results:

$$\begin{aligned}\Sigma F_y &= R_{Ay} + R_{By} - 30 \\ &= 12 + 18 - 30 \\ &= 0\end{aligned}$$

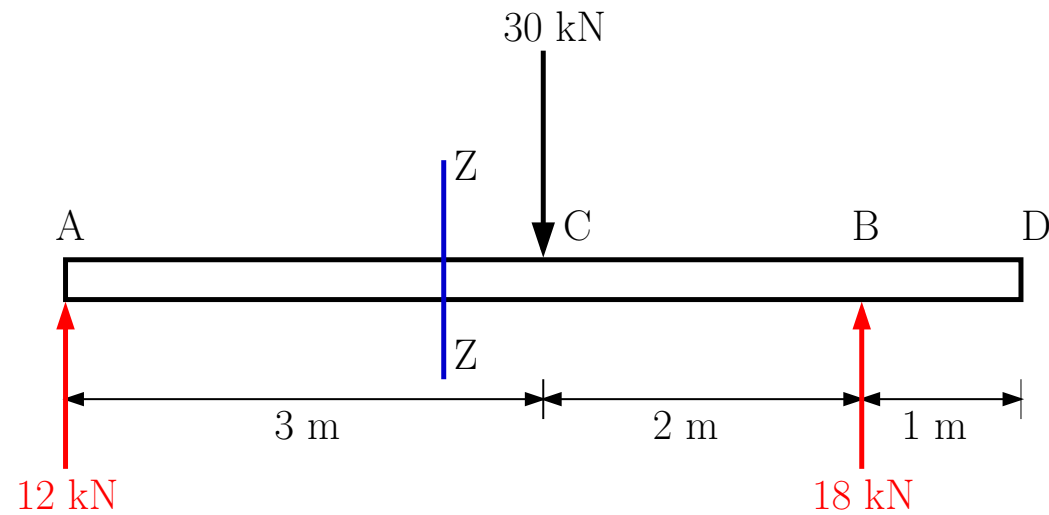
Thus,



Method of Diagrams

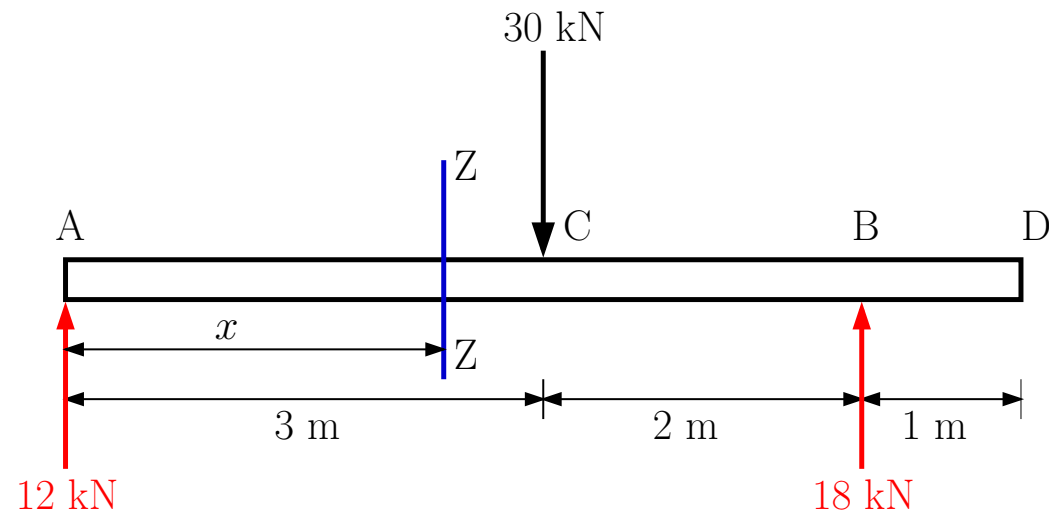


First, look at the segment from A to C :



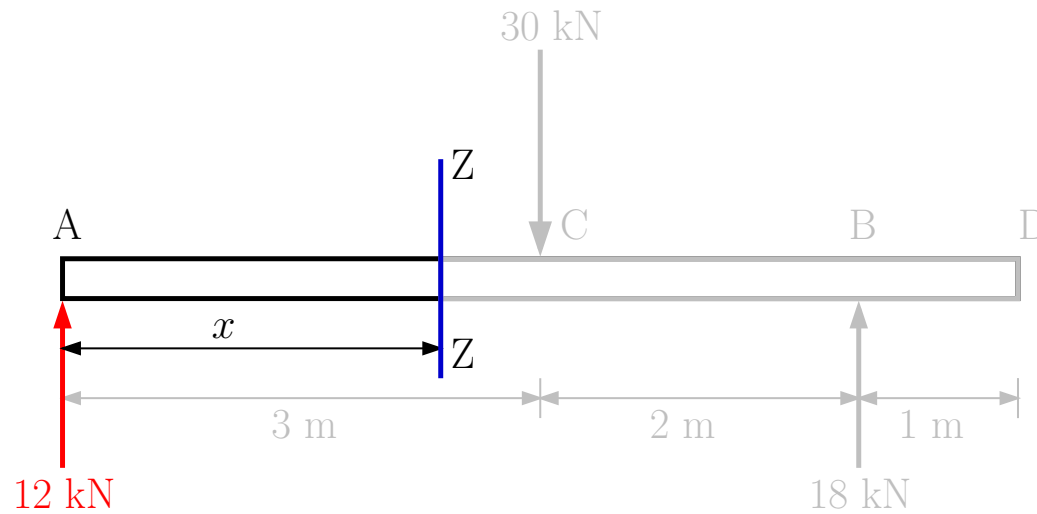
First, look at the segment from A to C :

- Draw a section ZZ through the beam between A and C



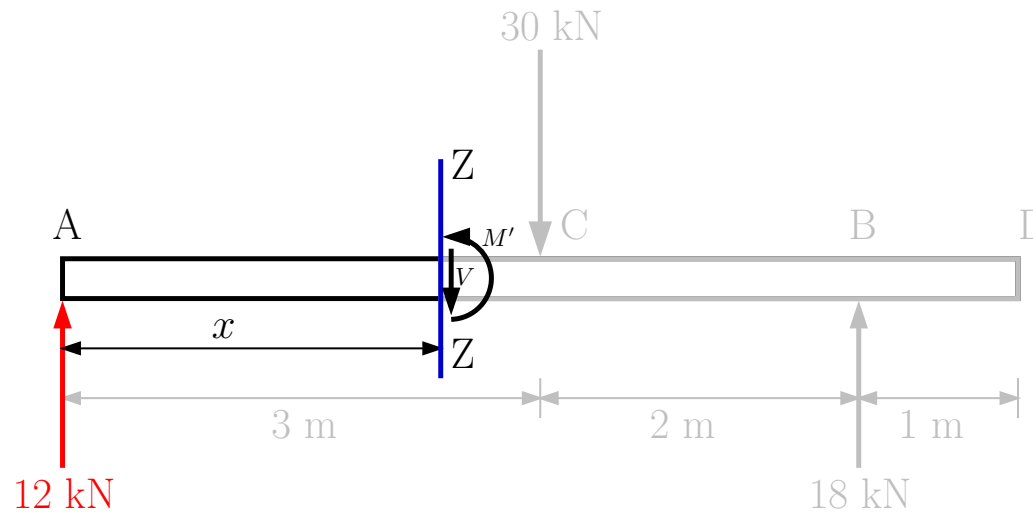
First, look at the segment from A to C :

- Draw a section ZZ through the beam between A and C
- Let ZZ be a distance x from A ; then $0 < x < 3$



First, look at the segment from A to C :

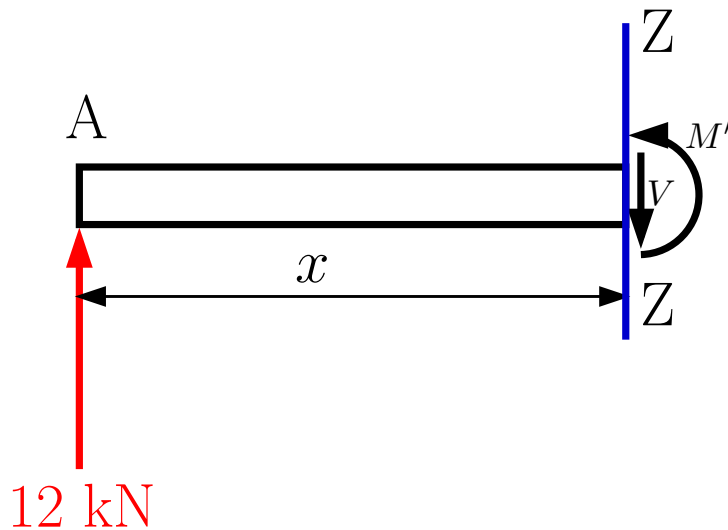
- Draw a section ZZ through the beam between A and C
- Let ZZ be a distance x from A ; then $0 < x < 3$
- Disregard the beam to the right of ZZ



First, look at the segment from A to C :

- Draw a section ZZ through the beam between A and C
- Let ZZ be a distance x from A ; then $0 < x < 3$
- Disregard the beam to the right of ZZ
- Consider the shear force V and the bending moment at ZZ to be positive

- To find the shear force V , sum the y components of forces between A and ZZ



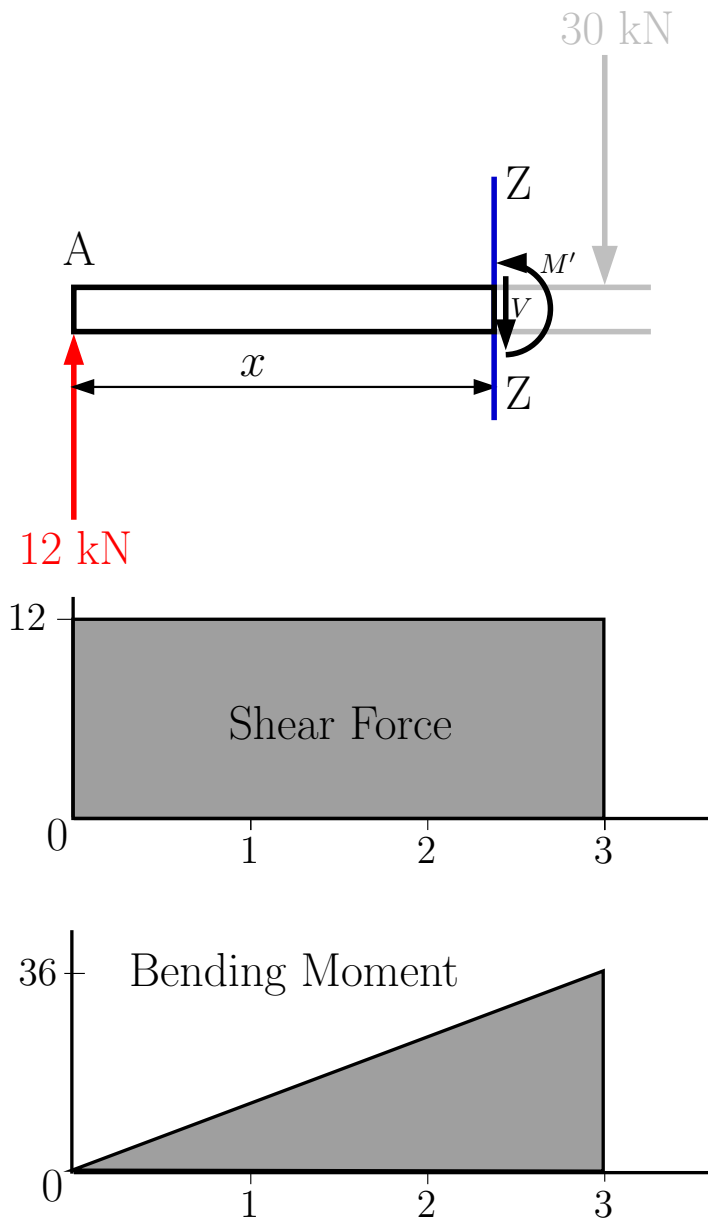
$$\Sigma F_y = 12 - V = 0$$

$$V = 12 \text{ kN}$$

- To find the bending moments, sum the moments about the section ZZ

$$\Sigma M_{ZZ} = M' - 12 \times x = 0$$

$$M' = 12x \text{ kN} \cdot \text{m}$$

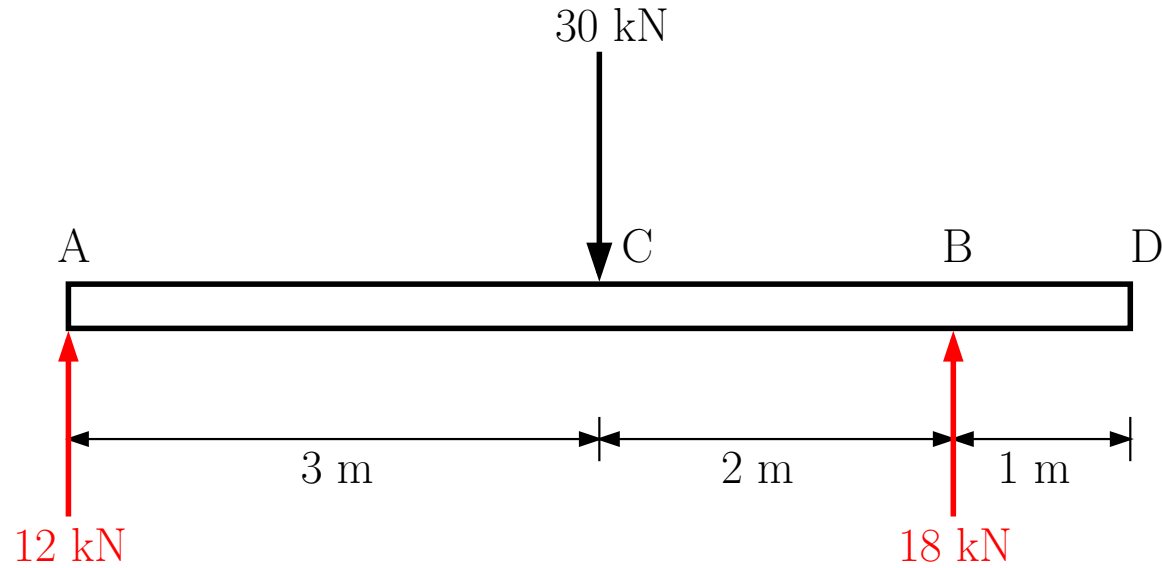


- $V = 12$ kN for all values of x such that $0 < x < 3$.

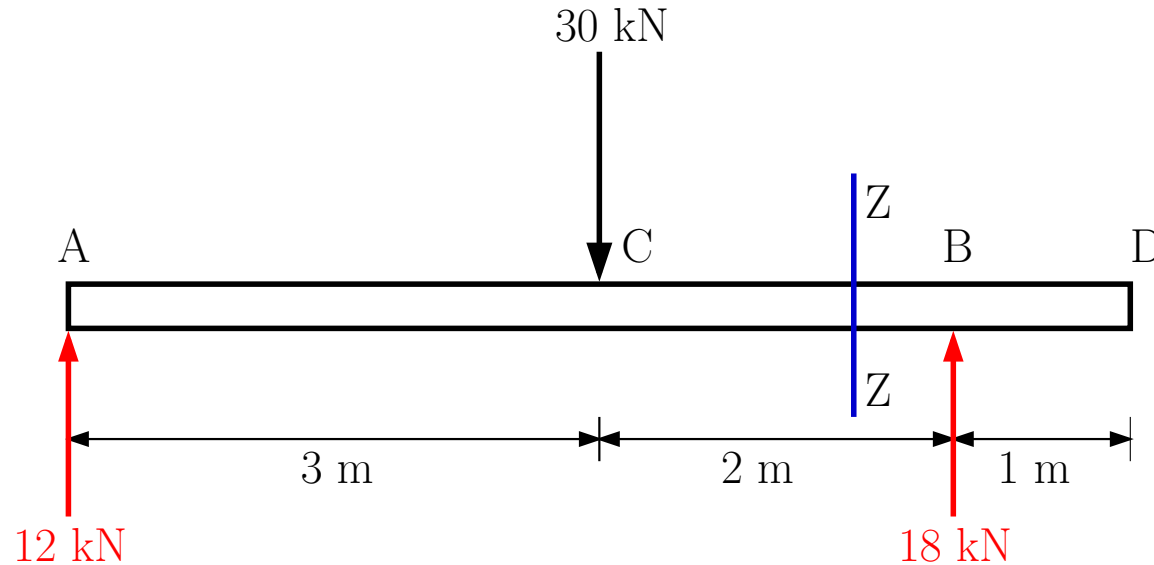
The shear force is **constant** over this range.

- $M' = 12x$ kN · m so the bending moment varies **linearly** with x

Method of Diagrams

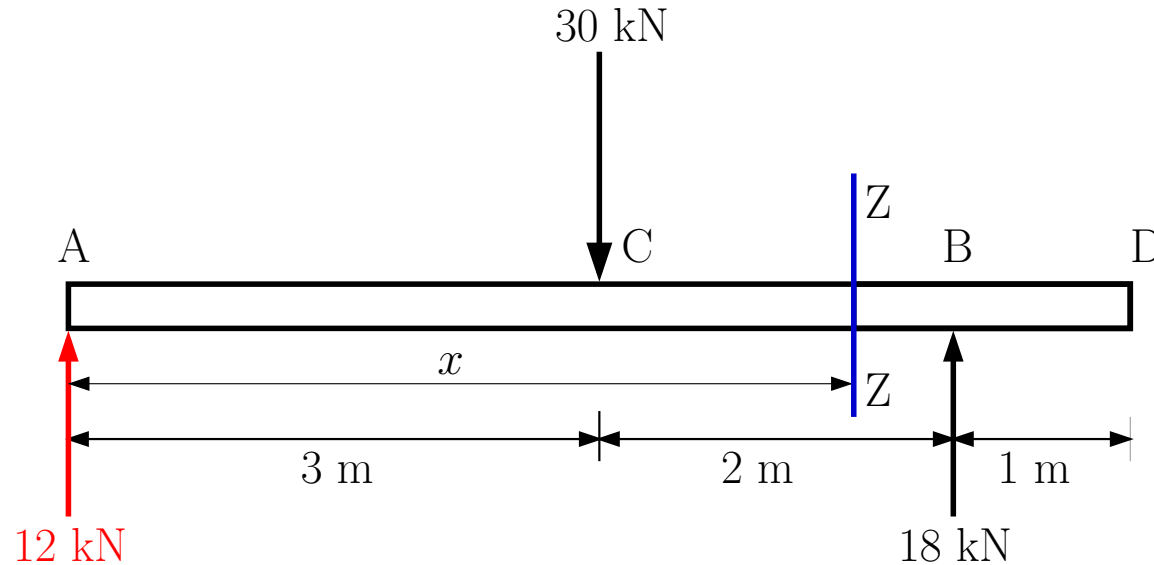


Now look at the segment from C to B :



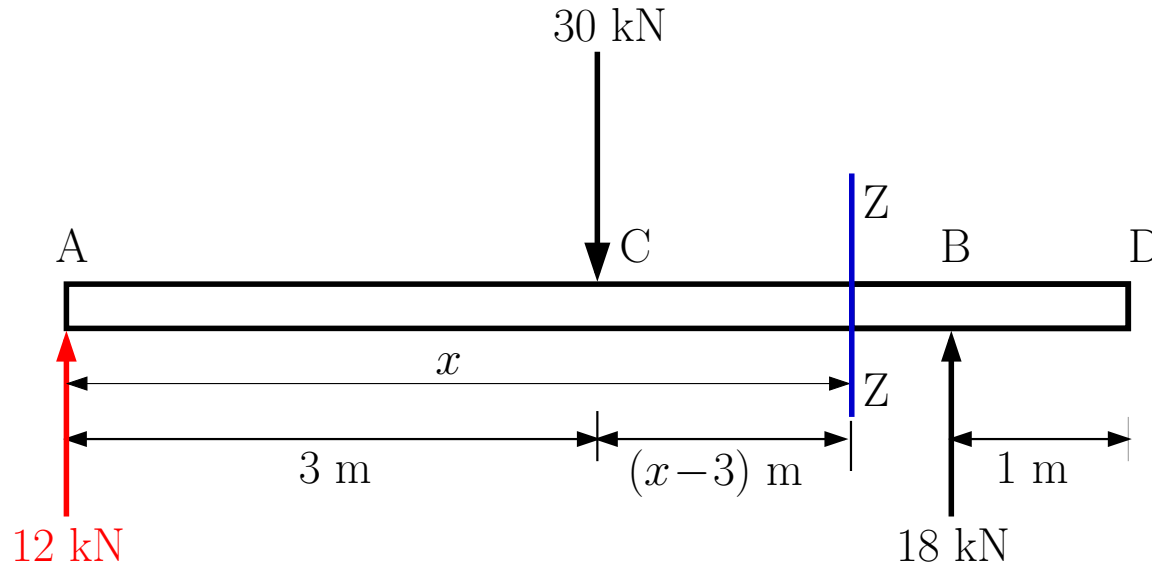
Now look at the segment from C to B :

- Draw a section ZZ through the beam between C and B



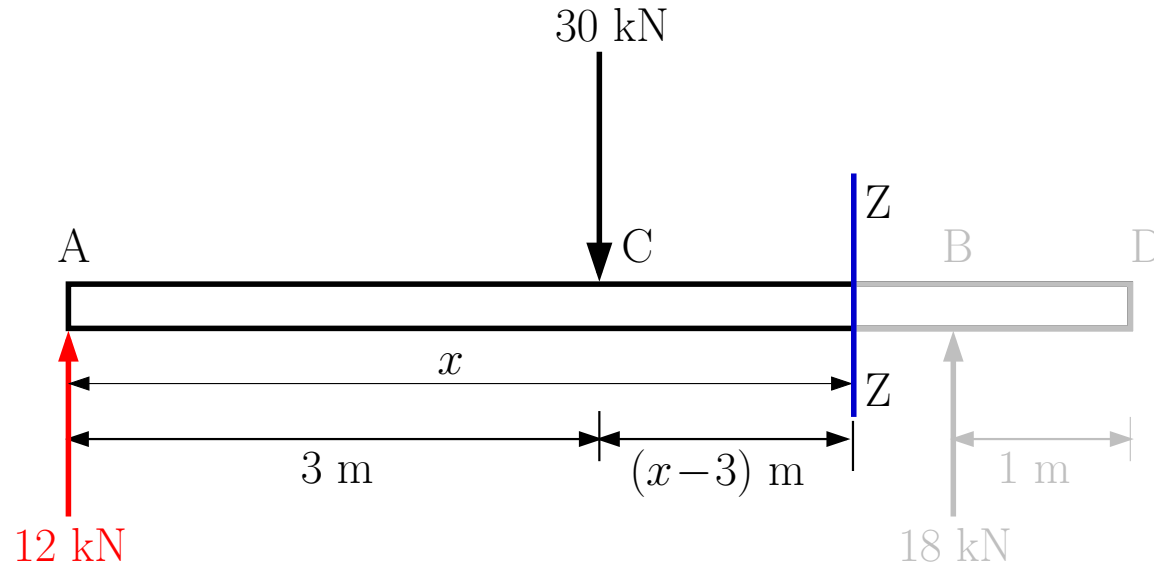
Now look at the segment from C to B :

- Draw a section ZZ through the beam between C and B
- Let ZZ be a distance x from A ; then $3 < x < 5$



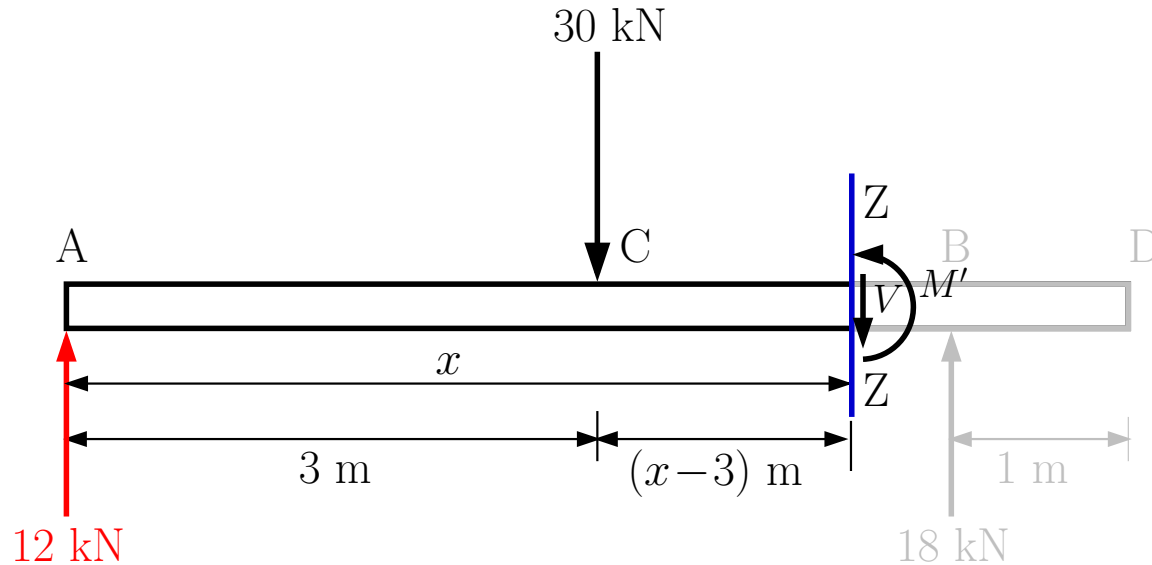
Now look at the segment from C to B :

- Draw a section ZZ through the beam between C and B
- Let ZZ be a distance x from A ; then $3 < x < 5$
- The distance from ZZ to C is $x - 3$



Now look at the segment from C to B :

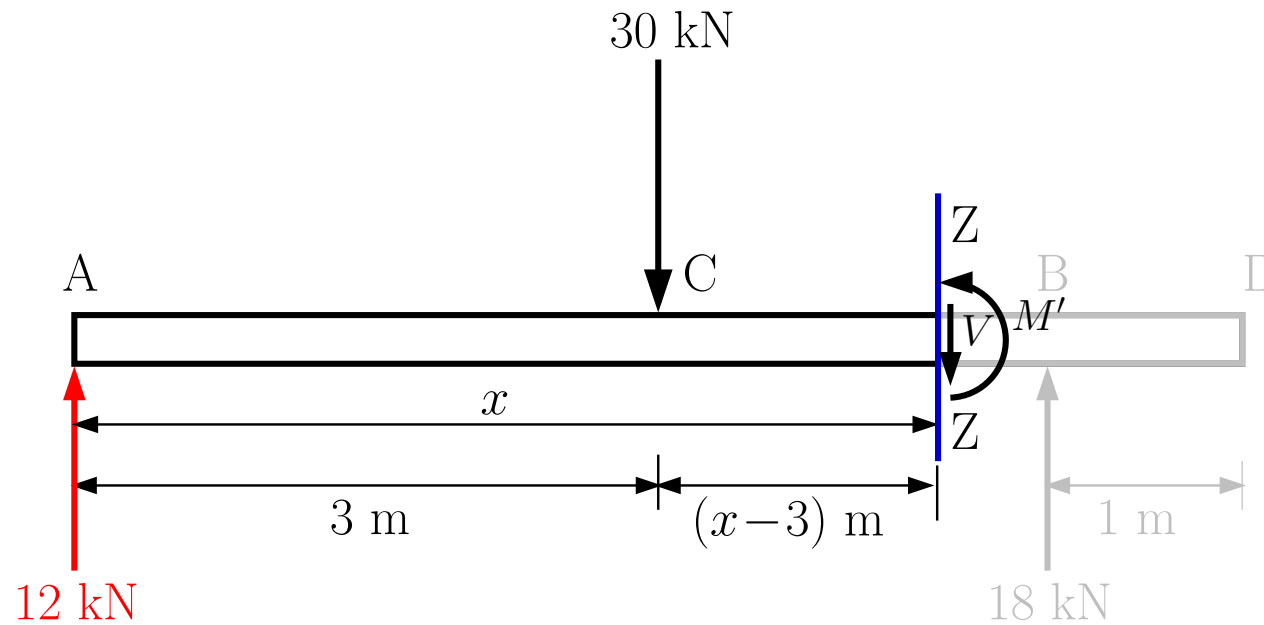
- Draw a section ZZ through the beam between C and B
- Let ZZ be a distance x from A ; then $3 < x < 5$
- The distance from ZZ to C is $x - 3$
- Disregard the beam to the right of ZZ



Now look at the segment from C to B :

- Draw a section ZZ through the beam between C and B
- Let ZZ be a distance x from A ; then $3 < x < 5$
- The distance from ZZ to C is $x - 3$
- Disregard the beam to the right of ZZ
- Consider the shear force V and the bending moment at ZZ to be positive

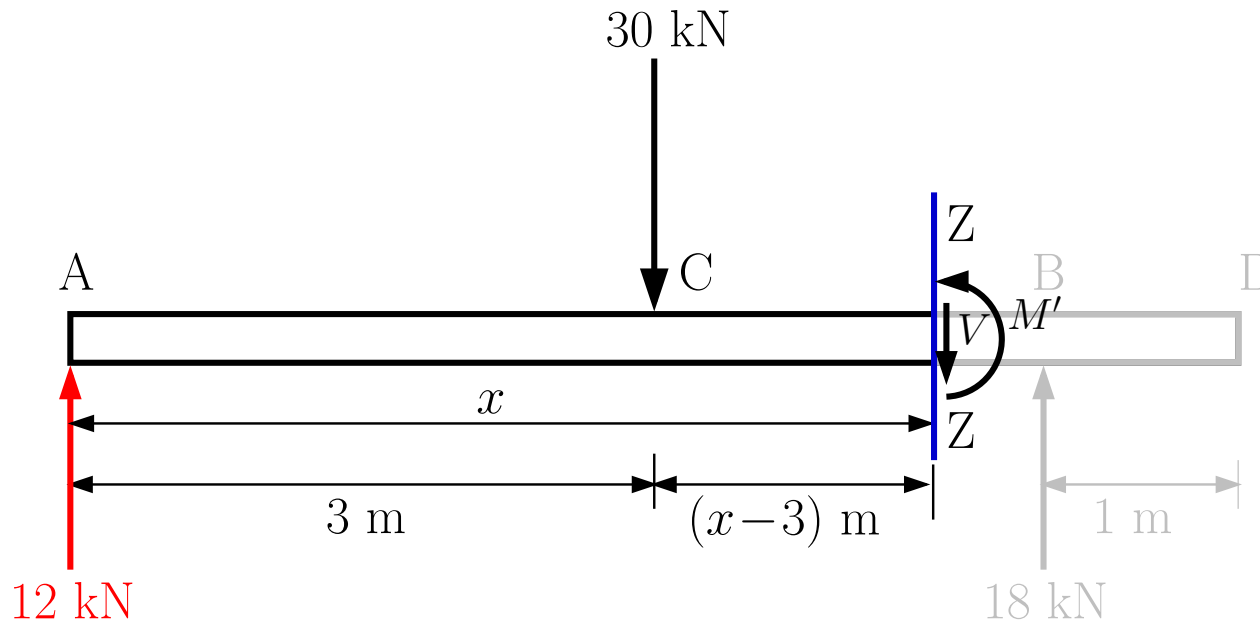
Method of Diagrams



$$\Sigma F_y = 12 - 30 - V = 0$$

$$V = -18 \text{ kN}$$

$$\text{We have } V = \begin{cases} 12 & \text{if } 0 < x < 3 \\ -18 & \text{if } 3 < x < 5 \end{cases}$$



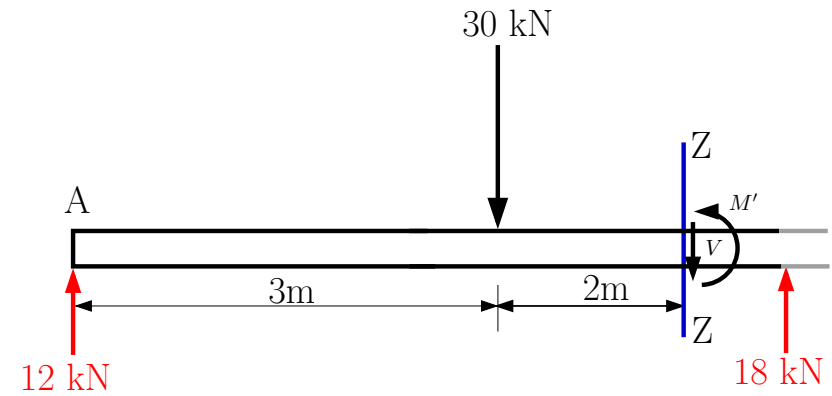
$$\Sigma M_{ZZ} = M' + 30(x - 3) - 12x = 0$$

$$M' = 12x - 30(x - 3)$$

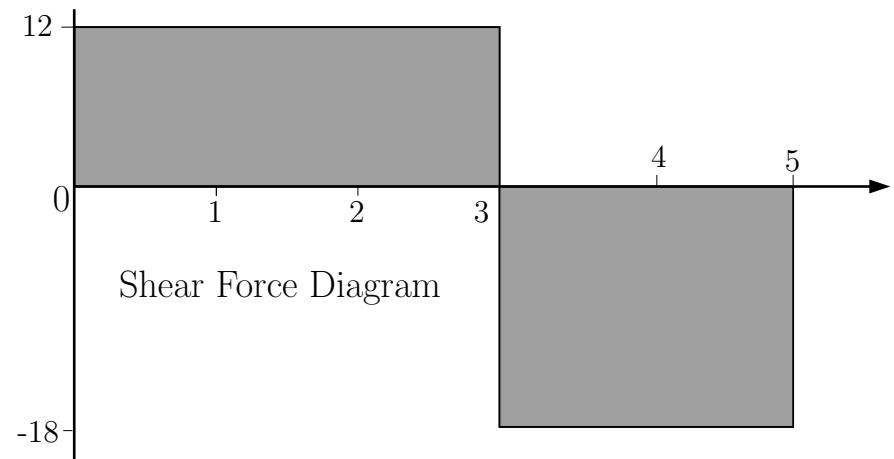
$$M' = 90 - 18x \text{ kN} \cdot \text{m}$$

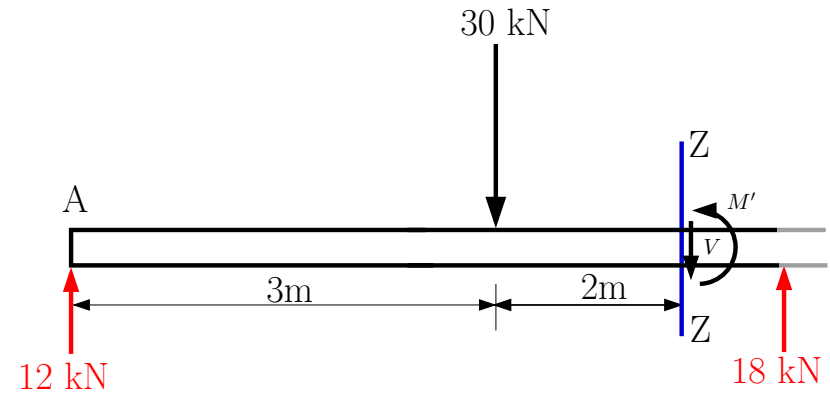
$$\text{We have } M' = \begin{cases} 12x & \text{if } 0 < x < 3 \\ 90 - 18x & \text{if } 3 < x < 5 \end{cases}$$

Method of Diagrams

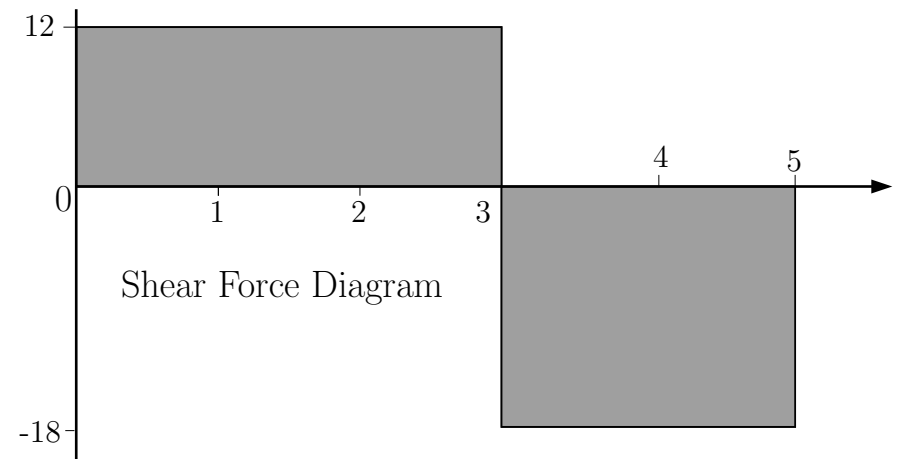


$$V = \begin{cases} 12 & \text{if } 0 < x < 3 \\ -18 & \text{if } 3 < x < 5 \end{cases}$$

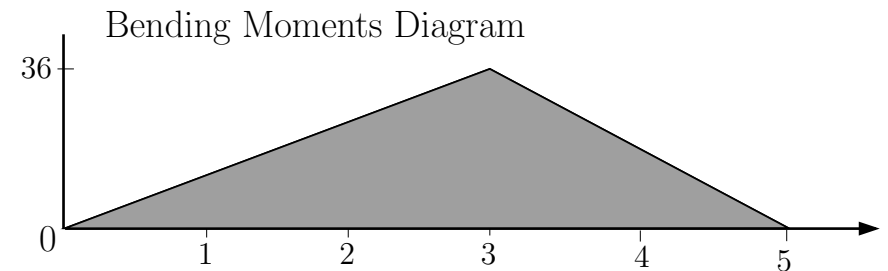




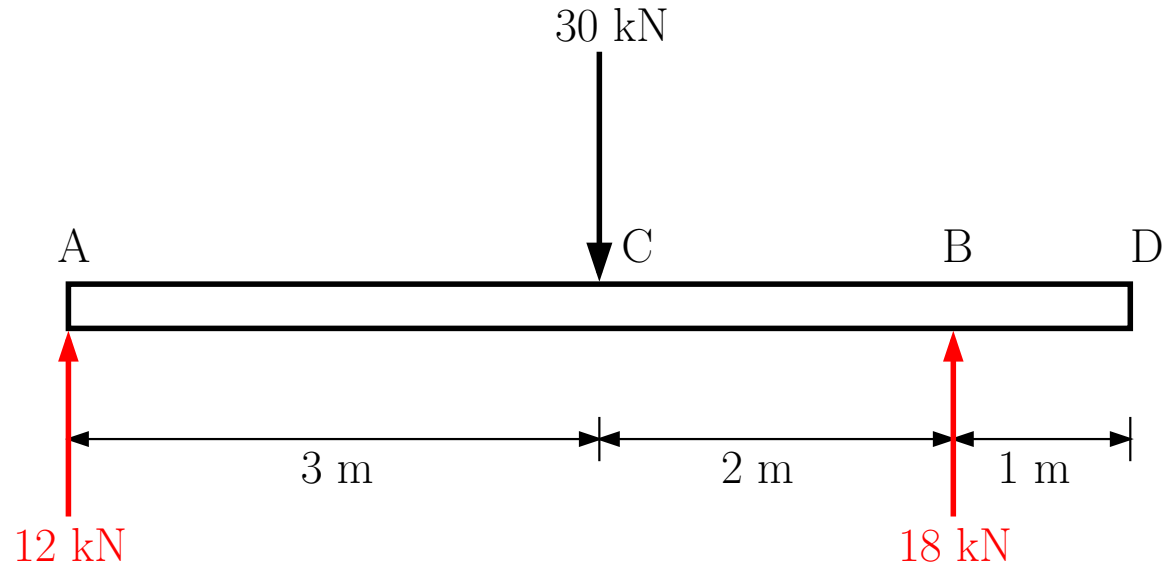
$$V = \begin{cases} 12 & \text{if } 0 < x < 3 \\ -18 & \text{if } 3 < x < 5 \end{cases}$$



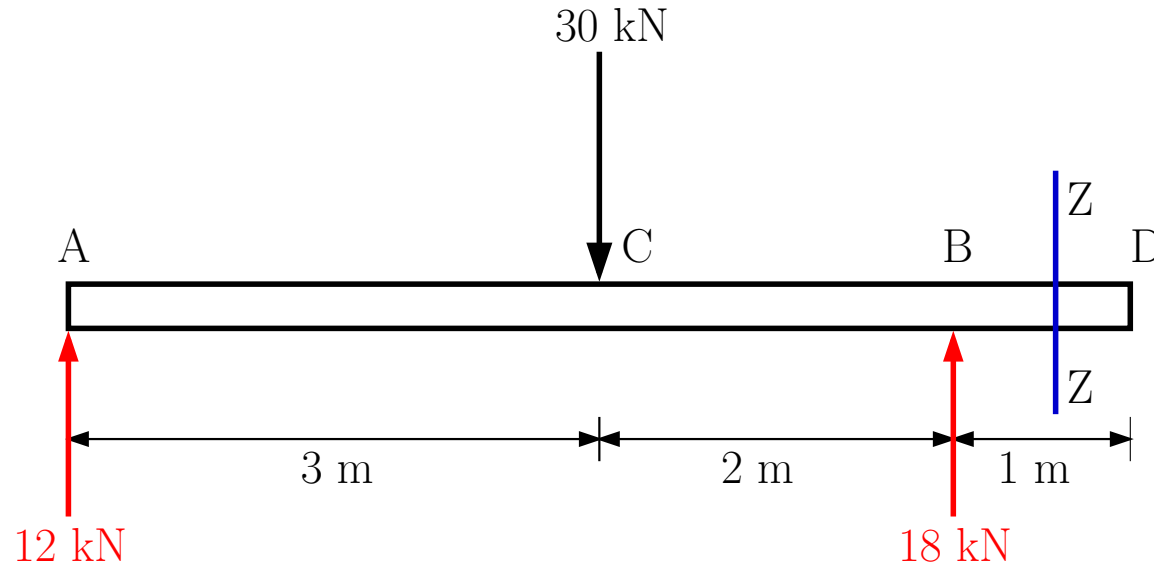
$$M' = \begin{cases} 12x & \text{if } 0 < x < 3 \\ 90 - 18x & \text{if } 3 < x < 5 \end{cases}$$



Method of Diagrams

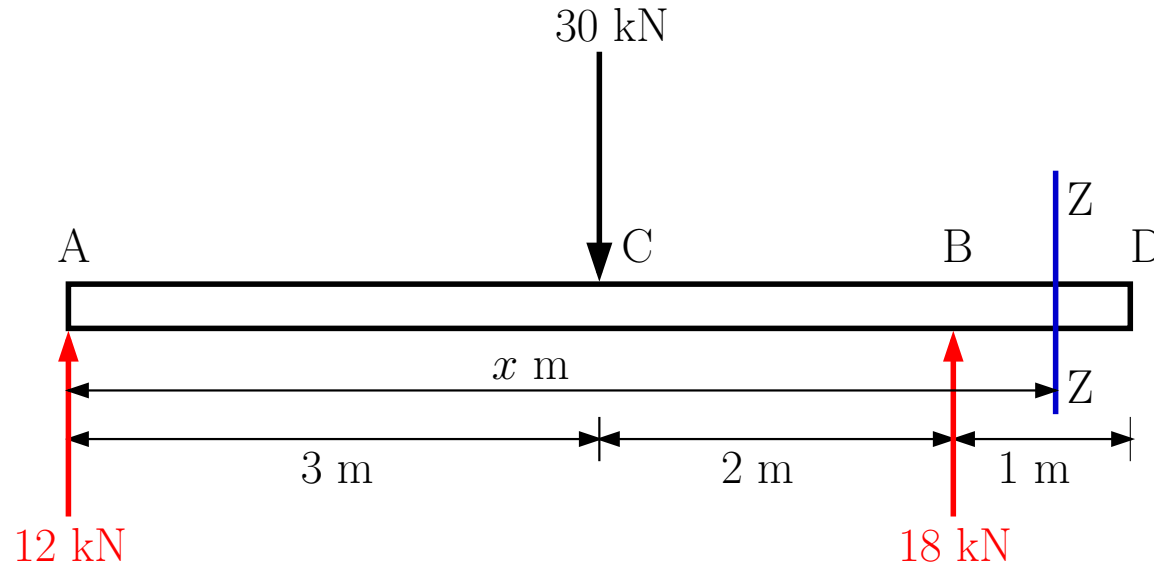


Now look at the segment from *B* to *D*:



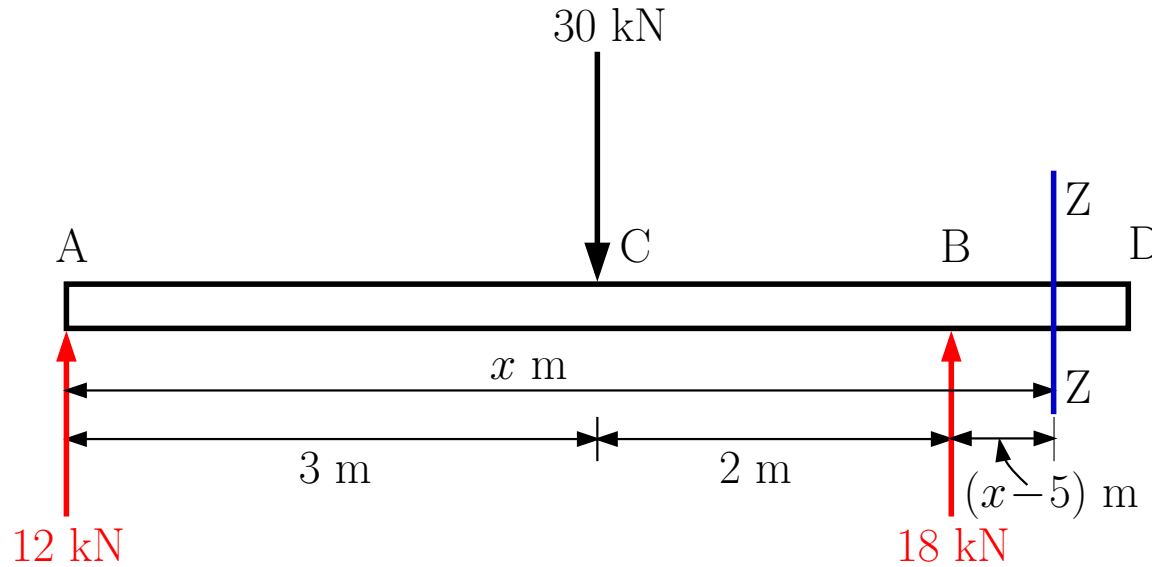
Now look at the segment from B to D :

- Draw a section ZZ through the beam between B and D



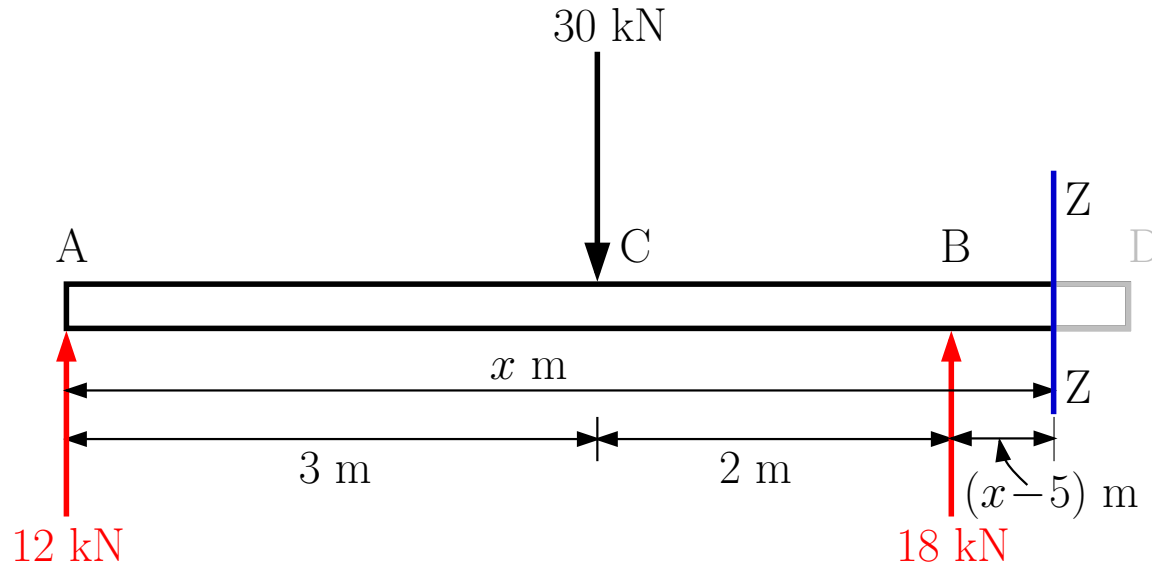
Now look at the segment from B to D :

- Draw a section ZZ through the beam between B and D
- Let ZZ be a distance x from A ; then $5 < x < 6$



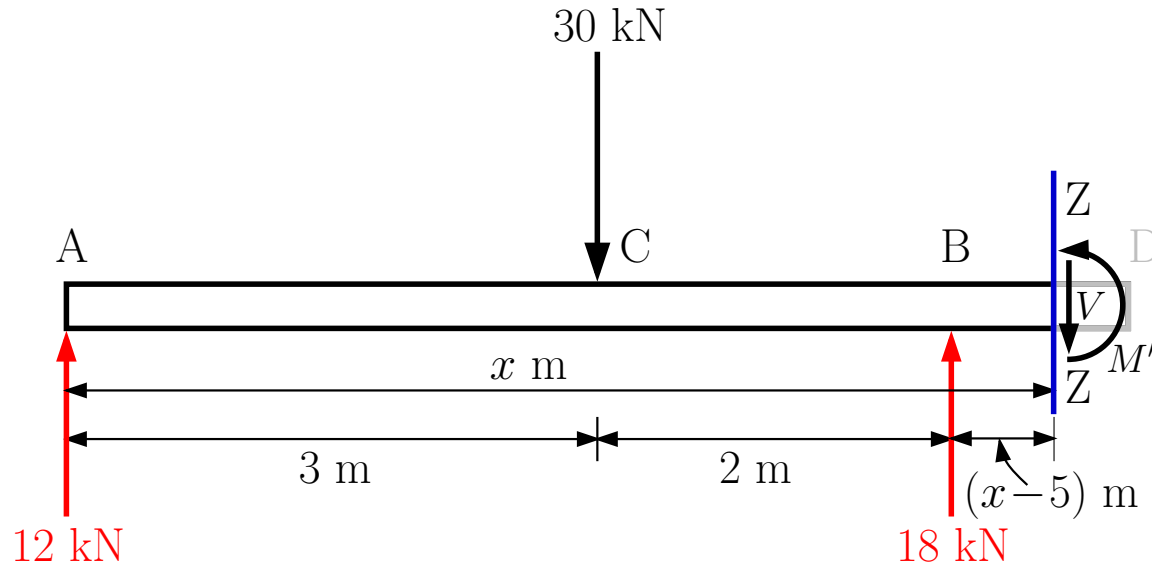
Now look at the segment from B to D :

- Draw a section ZZ through the beam between B and D
- Let ZZ be a distance x from A ; then $5 < x < 6$
- The distance from ZZ to B is $x - 5$



Now look at the segment from B to D :

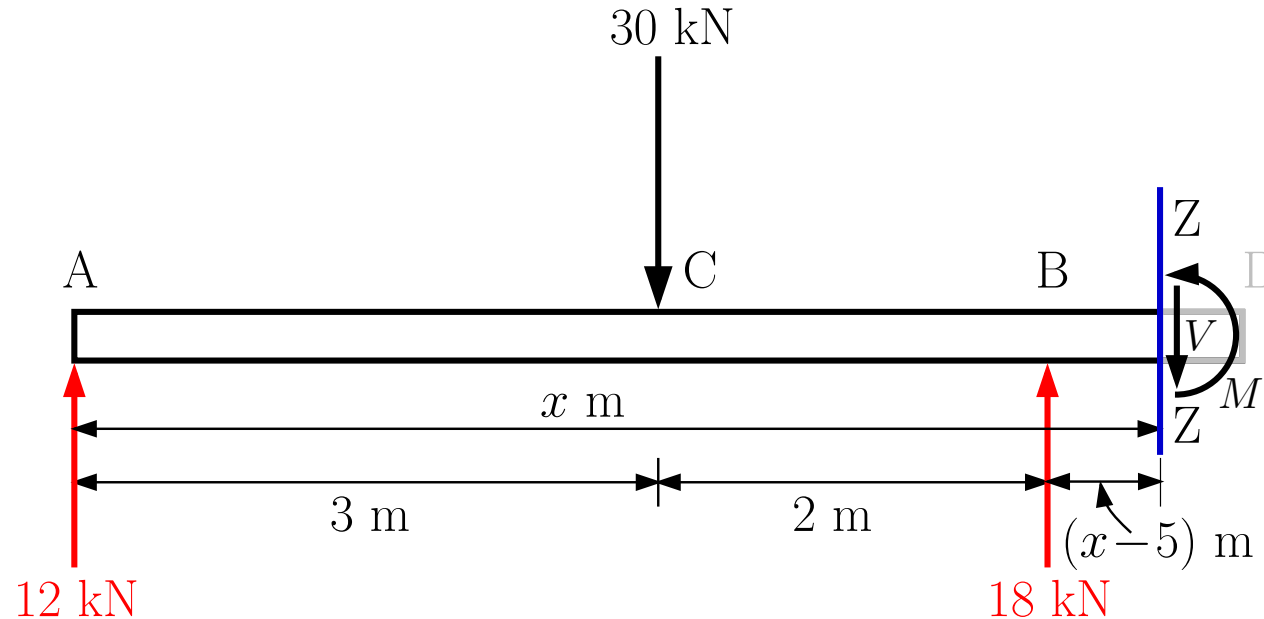
- Draw a section ZZ through the beam between B and D
- Let ZZ be a distance x from A ; then $5 < x < 6$
- The distance from ZZ to B is $x - 5$
- Disregard the beam to the right of ZZ



Now look at the segment from B to D :

- Draw a section ZZ through the beam between B and D
- Let ZZ be a distance x from A ; then $5 < x < 6$
- The distance from ZZ to B is $x - 5$
- Disregard the beam to the right of ZZ
- Consider the shear force V and the bending moment at ZZ to be positive

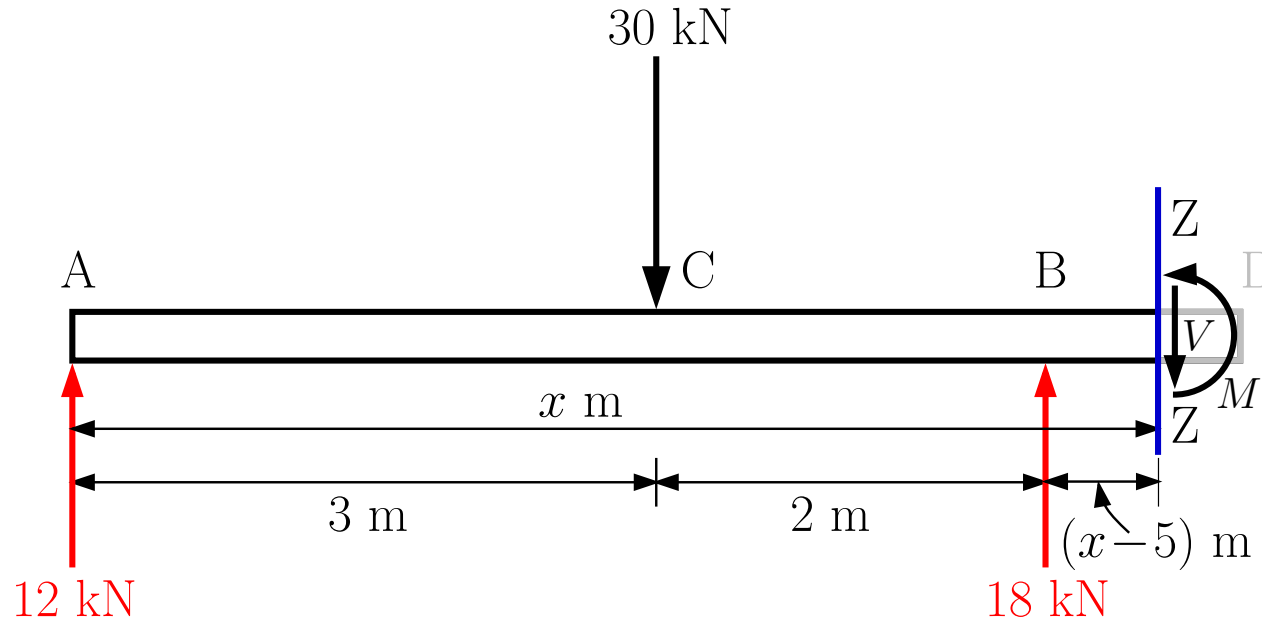
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$$\Sigma F_y = 12 - 30 + 18 - V = 0$$

$$V = 0 \text{ kN}$$

$$\text{We have } V = \begin{cases} 12 & \text{if } 0 < x < 3 \\ -18 & \text{if } 3 < x < 5 \\ 0 & \text{if } 5 < x \leq 6 \end{cases}$$



$$\Sigma M_{ZZ} = M' + 30(x - 5 + 2) - 12x - 18(x - 5) = 0$$

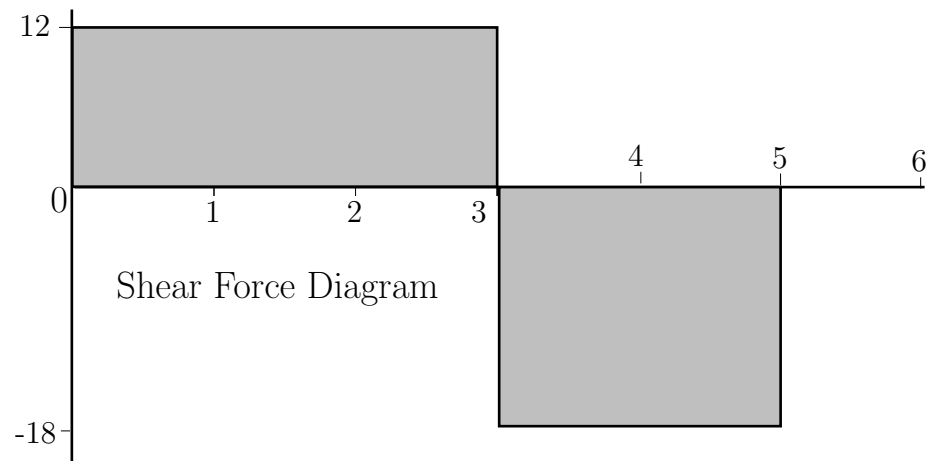
$$\Rightarrow M' + 30x - 90 - 12x - 18x + 90 = 0$$

$$M' = 0 \text{ kN} \cdot \text{m}$$

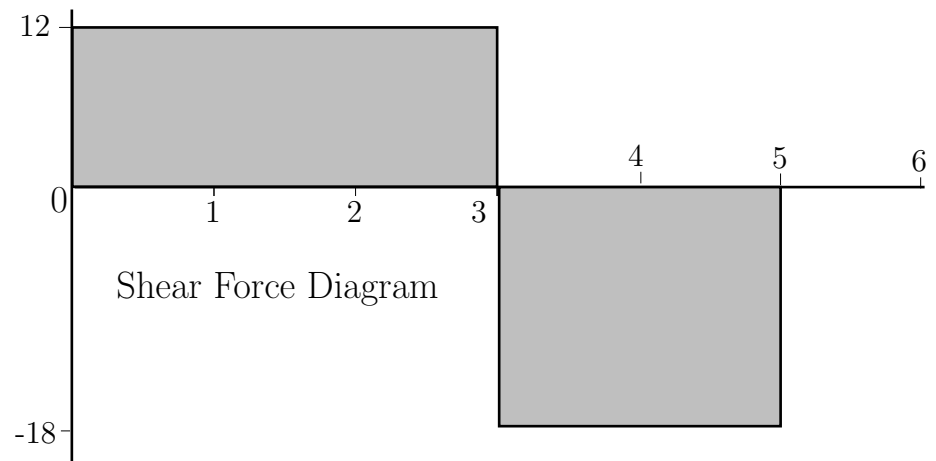
$$\text{We have } M' = \begin{cases} 12x & \text{if } 0 < x < 3 \\ 90 - 18x & \text{if } 3 < x < 5 \\ 0 & \text{if } 5 < x \leq 6 \end{cases}$$

Method of Diagrams

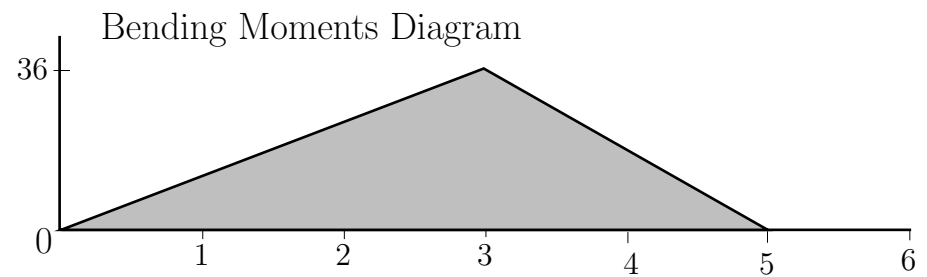
$$V = \begin{cases} 12 & \text{if } 0 < x < 3 \\ -18 & \text{if } 3 < x < 5 \\ 0 & \text{if } 5 < x \leq 6 \end{cases}$$



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$$M' = \begin{cases} 12x & \text{if } 0 < x < 3 \\ 90 - 18x & \text{if } 3 < x < 5 \\ 0 & \text{if } 5 < x \leq 6 \end{cases}$$



Exercise:

