Stress
(Normal Stress)

Strength of Materials

Stress

- when a structural member is subjected to an external load, internal resisting forces develop within the member to balance the external forces
  e.g. internal resisting forces in truss members
- the intensity of an internal force per unit area is called **stress**
- how much a material deforms and when it will fail is related to the amount of **stress** in the material

Units of Stress

- stress is measured as force per unit area
- in U.S. customary units, stress is measured in pounds per square inch (psi)
- we shall (mainly) use SI (international system) units (metric units)
- forces are measured in newtons and area in square metres
- by definition, one newton per square metre is equal to one pascal, so the units of stress are:

$$1\ \text{N/m}^2 = 1\ \text{Pa (pascal)}$$

Units of Stress

- areas used in strength of materials are often small, so we shall frequently use N and mm^2. Then,

$$
\begin{aligned}
1\ \text{MPa} &= 10^6\ \text{Pa} \\
&= \frac{10^6\ \text{N}}{\text{m}^2} \\
&= \frac{10^6\ \text{N}}{\left(10^3\ \text{mm}\right)^2} \\
&= \frac{10^6\ \text{N}}{10^6\ \text{mm}^2}
\end{aligned}
$$

$$1\ \text{MPa} = 1\ \text{N}/\text{mm}^2$$

Units of Stress

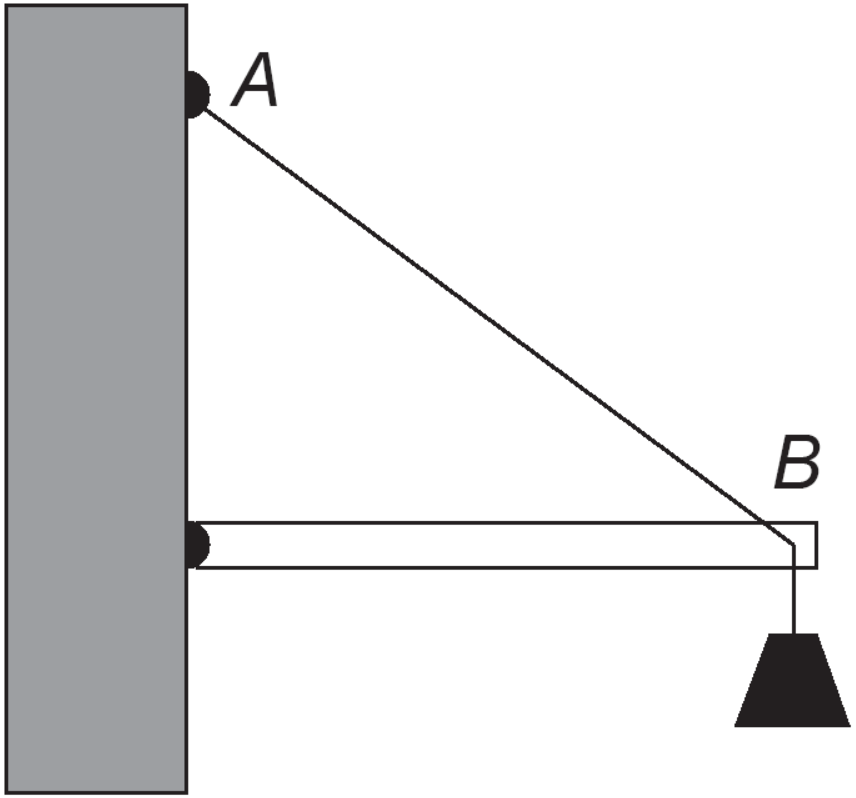
10^3 Pa	=	1 kPa	(kilopascal)
10^6 Pa	=	1 MPa	(megapascal)
10^9 Pa	=	1 GPa	(gigapascal)

Normal Stress

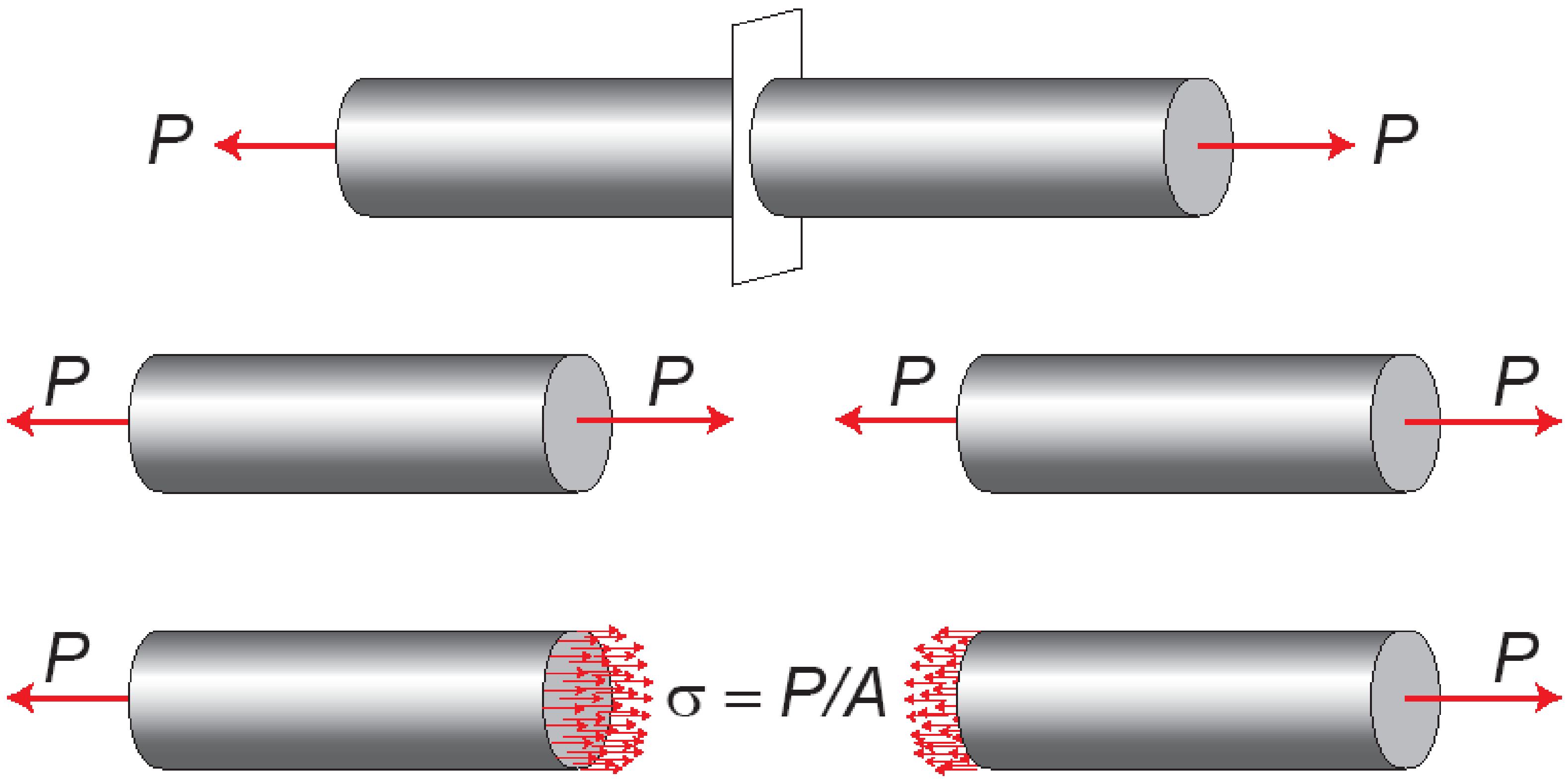
- normal stress is caused by internal forces that are **perpendicular** to the area considered
- tension in a cable is perpendicular to the cross-sectional area of the cable; this is **normal tensile stress**
- compression in a truss member is perpendicular to the cross-sectional area of the truss member; this is **normal compressive stress**

Normal Stress

- note that we refer to tension as a force
- normal stress is also known as **axial** stress since the tension is along the axis of the structural member (e.g. cable AB)
- cable AB is said to be **axially-loaded**



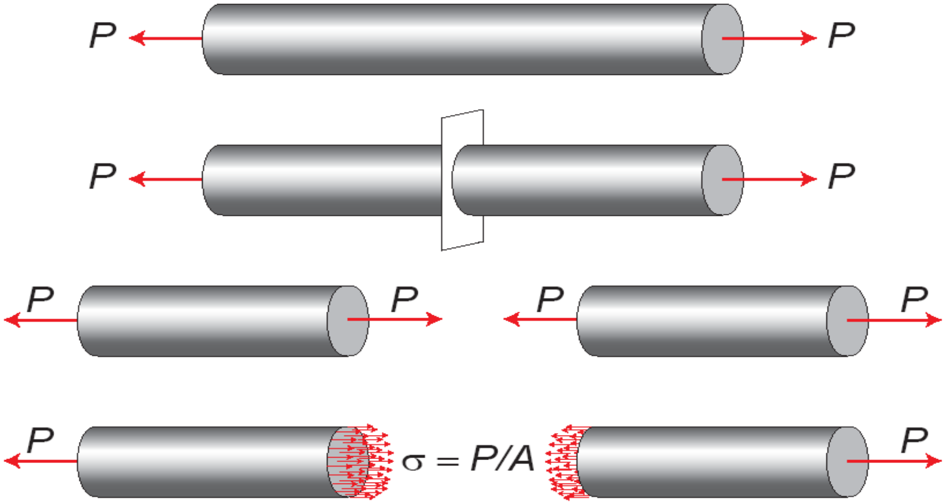
Normal Stress



Normal Stress

- *P* is the externally applied force
- *P* is equal to the internal resisting force at **any** section perpendicular to the axis of the structural member
- if *P* is applied through the centroid of the member, the normal stress, σ, is usually distributed uniformly over the cross-section and is given by the formula

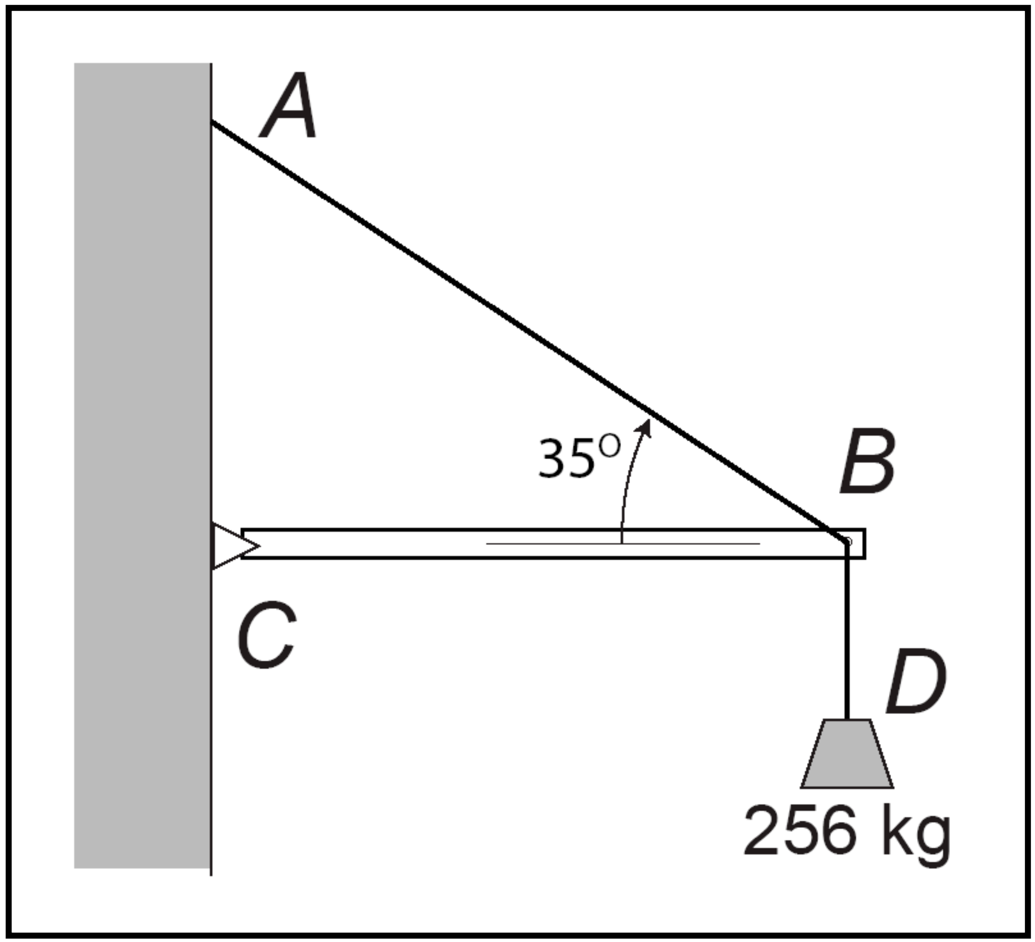
$$\sigma = P/A$$



- the Greek letter σ (sigma) denotes normal stress

Example:

Cable AB and cable BD have a diameter of 8.0 mm.
Calculate the normal stress in AB and in BD

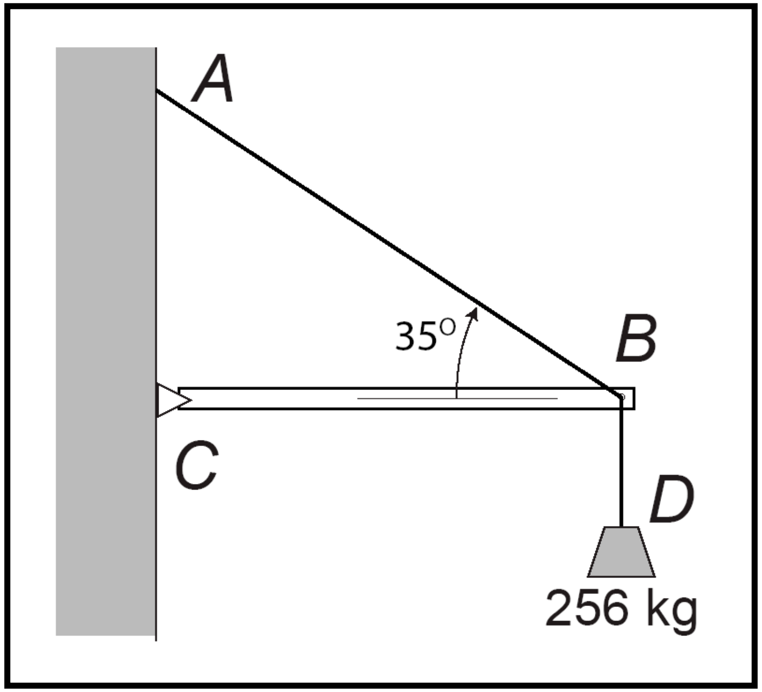


Example:

The tension in BD is easy to calculate since all the forces are vertical:

$$\Sigma F_y = T_{BD} - 256 \times 9.81 = 0$$

$$T_{BD} = 2511.4 \text{ N}$$



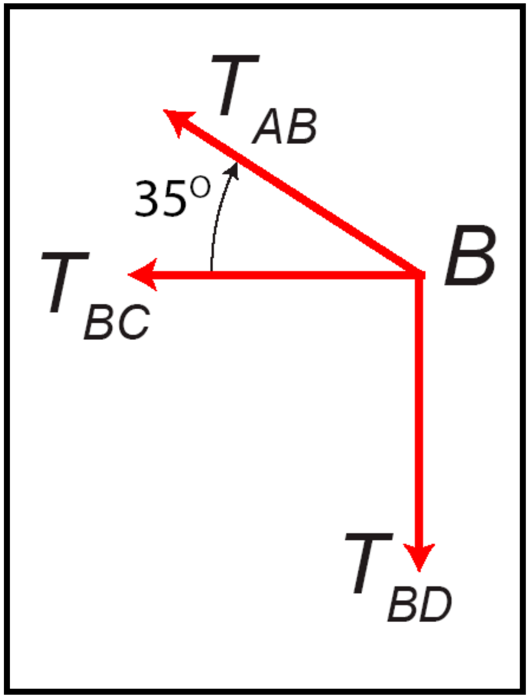
$$\sigma_{BD} = P/A$$

$$= \frac{2511.4 \text{ N}}{\pi \cdot (8.0 \text{ mm})^2/4}$$

$$= 49.963 \text{ MPa}$$

Example:

To find the tension in AB, consider the free body diagram of the concurrent forces at B:

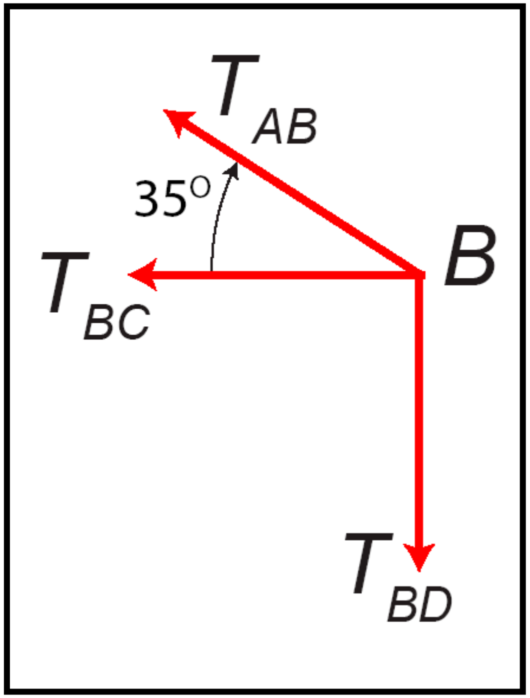


Example:

Then, summing the *y*-components:

$$\Sigma F_y = T_{AB} \sin 35^\circ - T_{BD} = 0$$

$$T_{AB} = \frac{2511.4}{\sin 35^\circ} = 4378.5 \text{ N}$$



$$\sigma_{AB} = \frac{4378.5 \text{ N}}{\pi \cdot (8.0 \text{ mm})^2 / 4} = 87.107 \text{ MPa}$$

Allowable Axial Force

- structural members are designed for a maximum **allowable stress**
- the maximum allowable load can be easily derived from the formula for normal stress:

$$\sigma_{allow} = P_{allow}/A$$
$$P_{allow} = \sigma_{allow} \cdot A$$

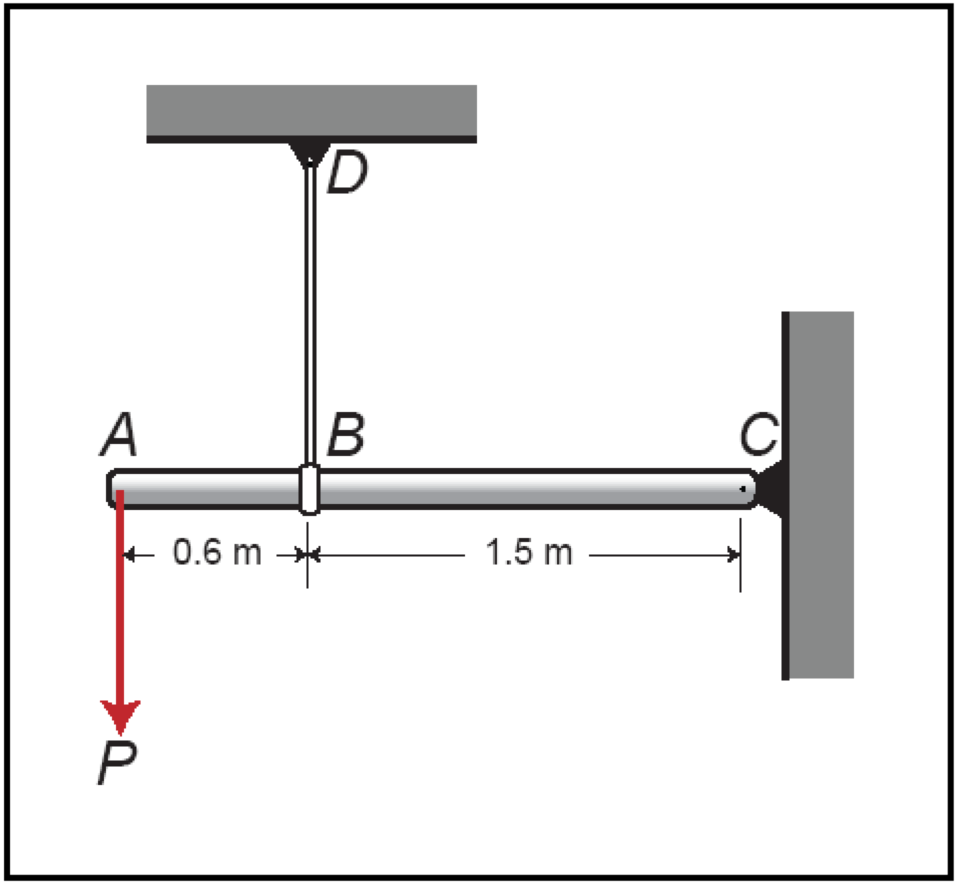
- then P_{allow} is the allowable axial load that a structural member with cross-sectional area A can carry without being overstressed

Example:

A rigid beam AC carries a load P and is supported by a pinned connection at C and a tie-rod BD.
σ_{allow} for the tie-rod BD is 145 MPa

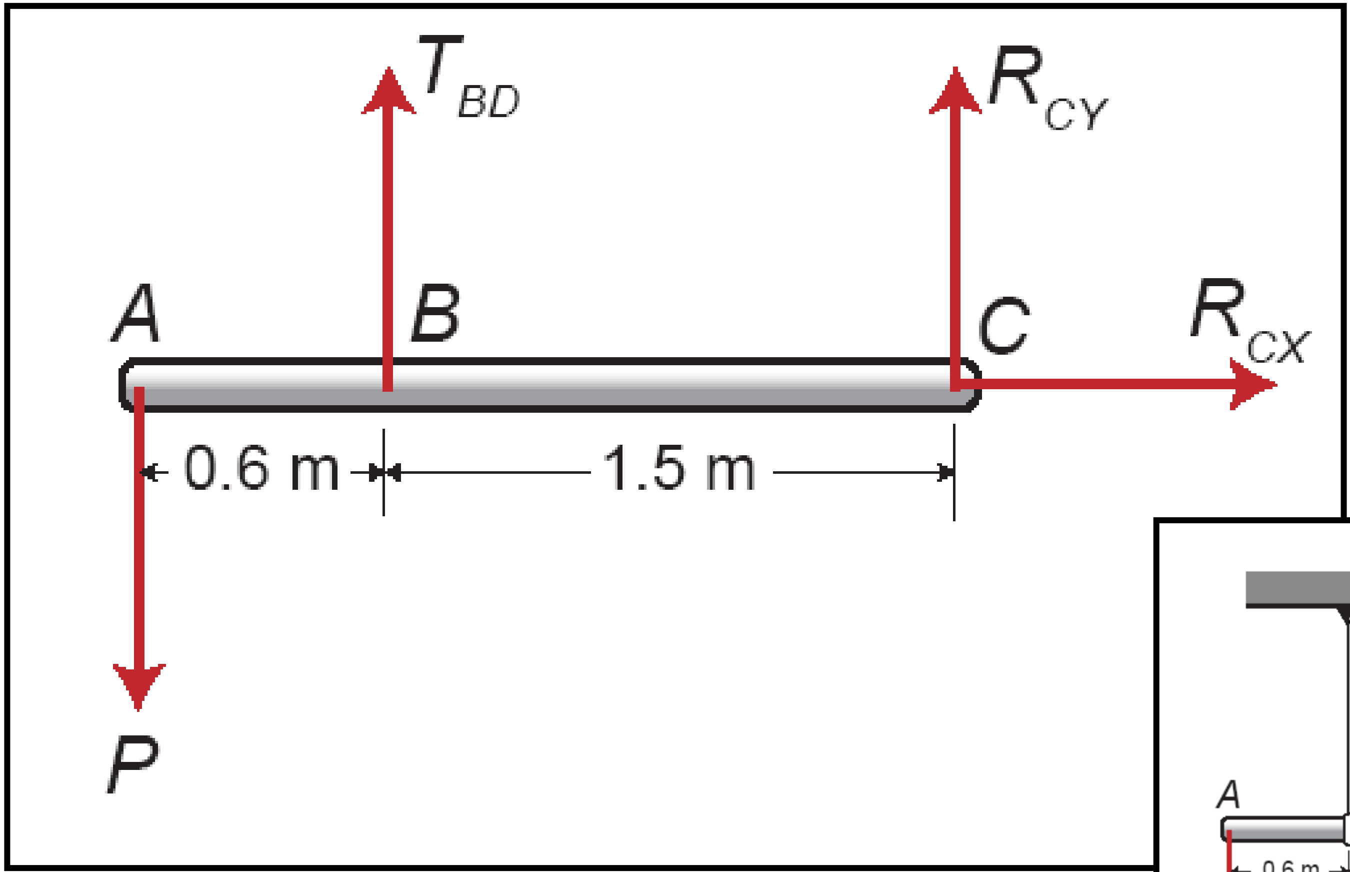
1. Find the maximum load P that can be supported if the tie rod has a cross-section of 25 mm x 25 mm

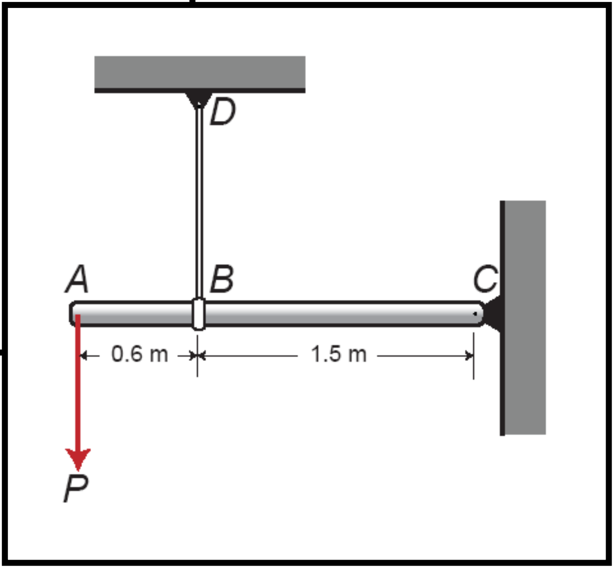
2. Find the diameter of a tie rod required to support 85 kN



Example:

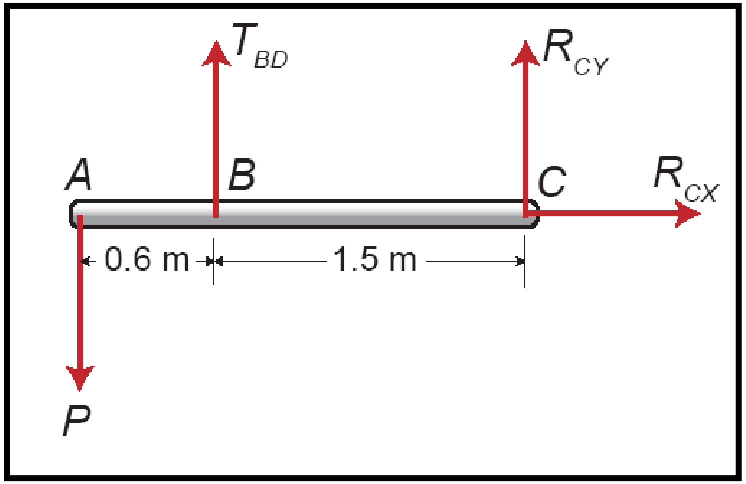
Draw the FBD of the beam:





Example:

Maximum Allowable Load:



Take moments about *C*

$$\Sigma M_C = P(2.1) - T_{BD}(1.5) = 0$$

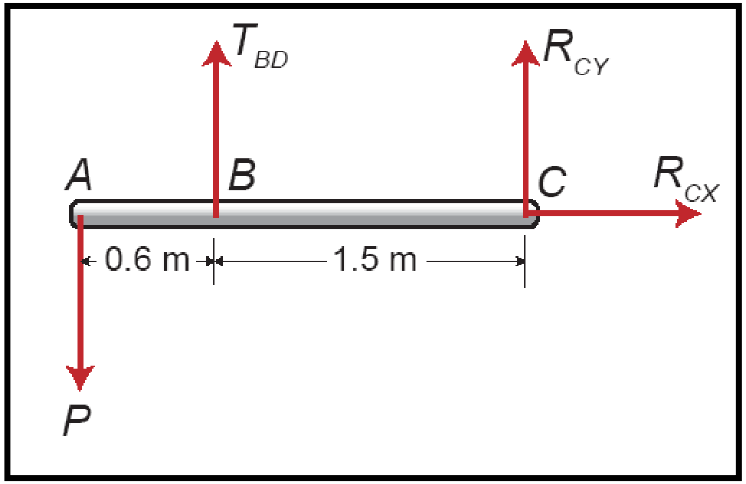
$$P = \frac{1.5}{2.1} \cdot T_{BD}$$

$$
\begin{aligned}
T_{BD_{allow}} &= \sigma_{allow} \cdot A \\
&= \left(145 \text{ N/mm}^2\right) \cdot (25 \text{ mm} \times 25 \text{ mm}) \\
&= 90625.0 \text{ N}
\end{aligned}
$$

$$
\begin{aligned}
P &= \frac{1.5}{2.1} \cdot (90.625) \text{ kN} \\
&= 64.732 \text{ kN}
\end{aligned}
$$

Example:

Required Diameter of Tie-Rod:



For a load of 85 kN, the tensile force in the tie-rod is given by:

$$T_{BD} = \frac{2.1}{1.5} \cdot P = \frac{2.1}{1.5} \cdot (85) = 119 \text{ kN}$$

Required cross-sectional area of the tie-rod is given by:

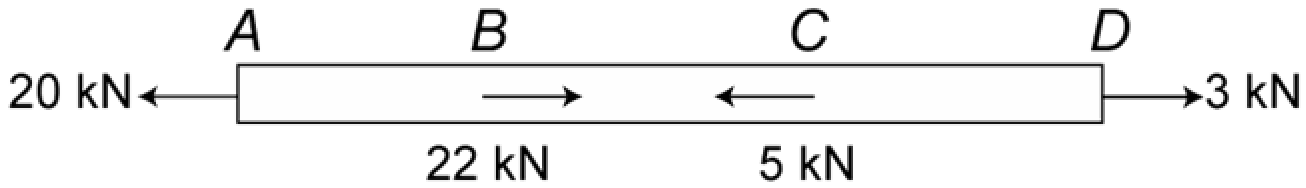
$$A = \frac{T_{BD}}{\sigma_{allow}} = \frac{119000 \text{ N}}{145 \text{ N/mm}^2} = 820.69 \text{ mm}^2$$

$$A = \pi d^2/4$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 820.69}{\pi}} = \sqrt{1044.9 \text{ mm}^2} = 32.325 \text{ mm}$$

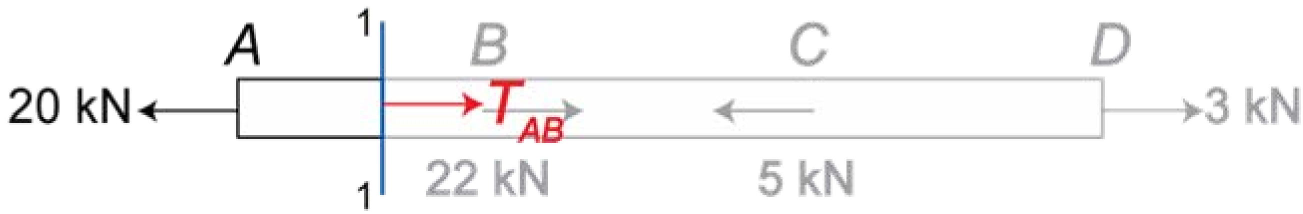
Internal Axial Force Diagram

Used when axial force varies along the length of a member:



1. The whole bar is in equilibrium since $\Sigma F_x = 0$
2. Any segment in the bar must also be in equilibrium
3. Draw a section 1-1 anywhere through segment AB and perendicular to the line of action of the axial force to find the internal force; repeat with sections 2-2 and 3-3 through segments BC and CD
4. Draw the internal axial force diagram

Internal Axial Force Diagram

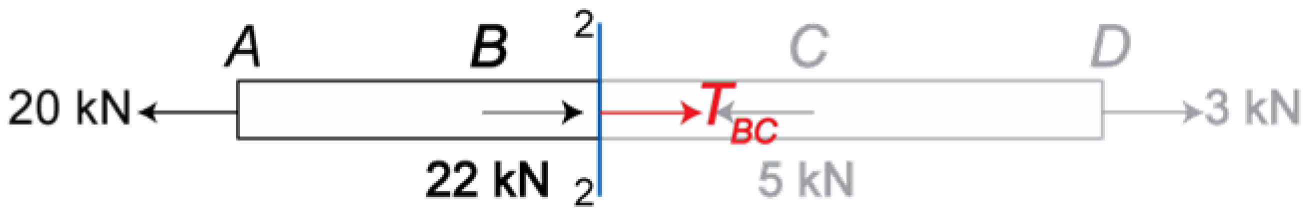


1. Draw a section 1-1 through segment *AB*
2. Consider the segment to the left of 1-1; the segment is in equilibrium so the internal force T_{AB} must be equal to the externally applied force at *A*

$$\Sigma F_x = T_{AB} - 20 = 0$$
$$T_{AB} = 20 \text{ kN}$$

3. The internal axial force in segment *AB* is 20 kN

Internal Axial Force Diagram

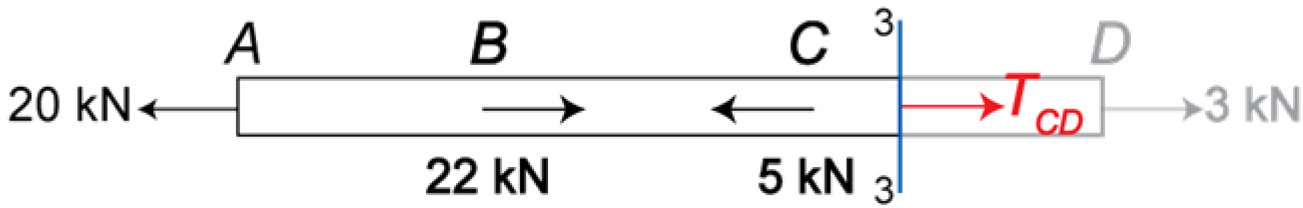


1. Draw a section 2-2 through segment *BC*
2. Consider the segment to the left of 2-2; the segment is in equilibrium so:

$$\begin{aligned} \Sigma F_x = T_{BC} + 22 - 20 &= 0 \\ T_{AB} &= -2 \text{ kN} \end{aligned}$$

3. The internal axial force in segment *BC* is -2 kN (i.e. the segment *BC* is in compression)

Internal Axial Force Diagram



1. Draw a section 3-3 through segment *CD*
2. Consider the segment to the left of 3-3; the segment is in equilibrium so:

$$\Sigma F_x = T_{CD} + 22 - 20 - 5 = 0$$
$$T_{CD} = 3 \text{ kN}$$

3. The internal axial force in segment *CD* is 3 kN

   - or consider the segment to the right of 3-3. Then, T_{CD} is out of the section, towards the left

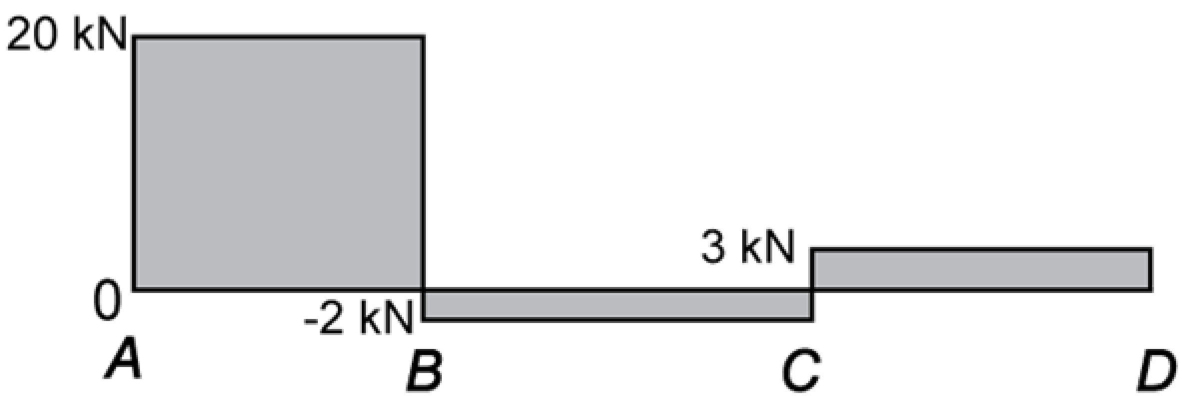
$$\Sigma F_x = 3 - T_{CD} = 0 \text{ so } T_{CD} = 3 \text{ kN (as before)}$$

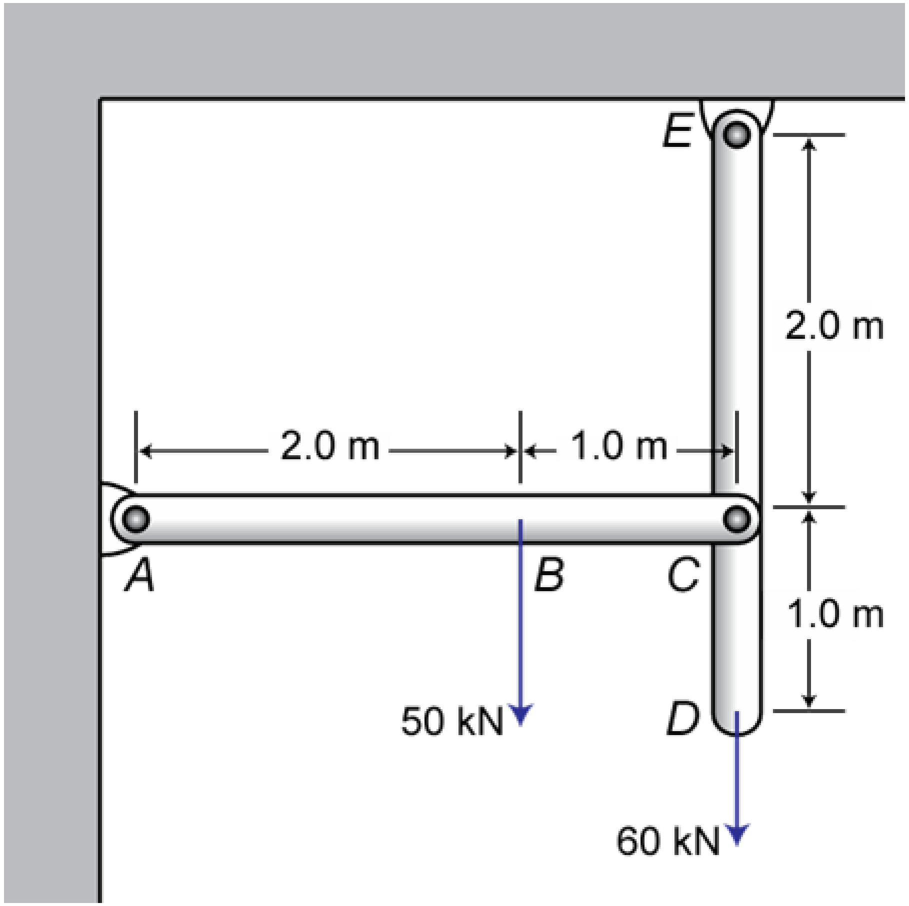
Internal Axial Force Diagram

We have:

$$T_{AB} = 20 \text{ kN}$$
$$T_{BC} = -2 \text{ kN}$$
$$T_{CD} = 3 \text{ kN}$$

The internal axial force diagram is drawn with tensile forces above the 0 line and compressive forces below:



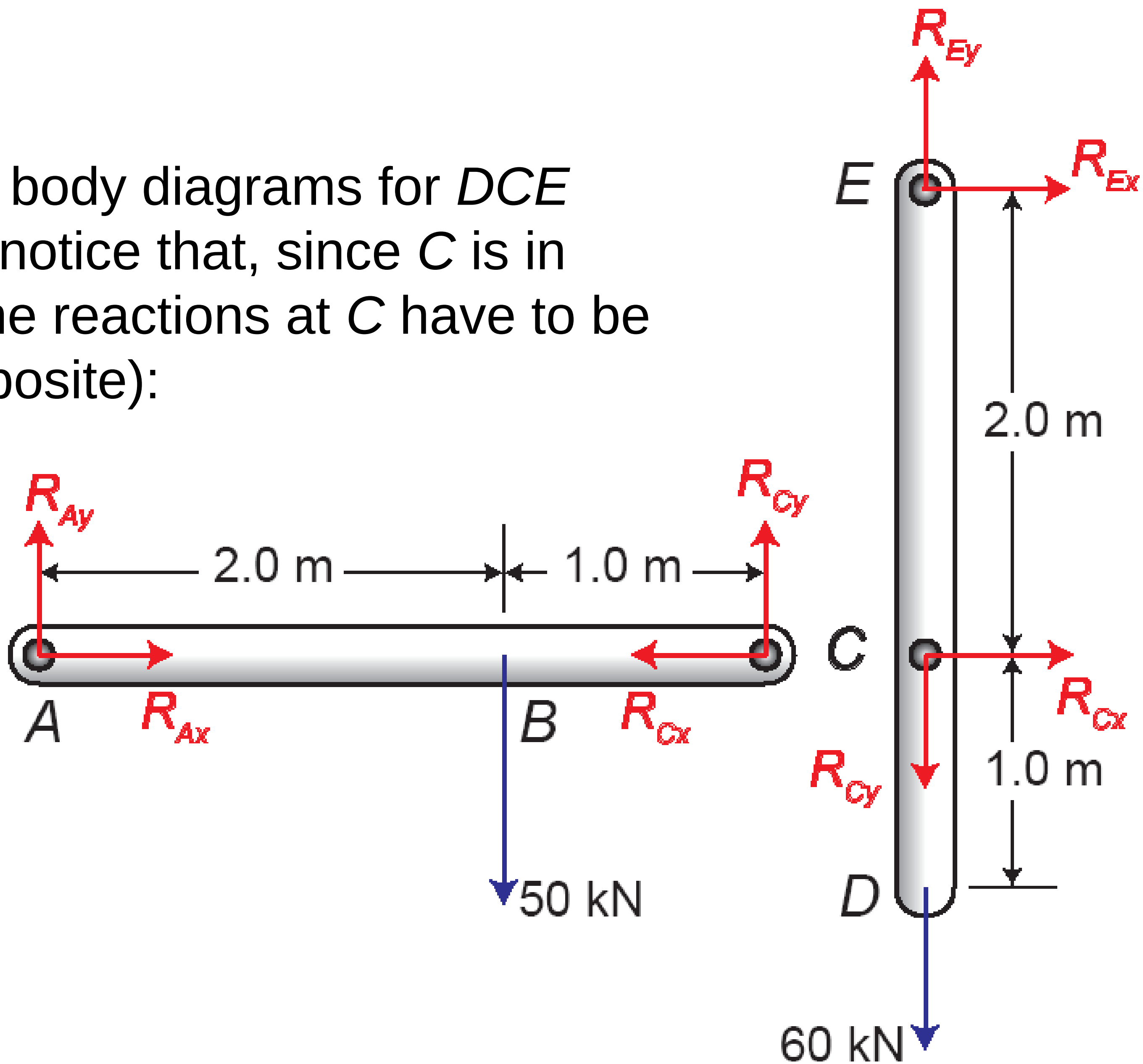


Example:

Connections at *A, C* and *E* are pinned. Draw the internal axial force diagram for member *DCE*

Example:

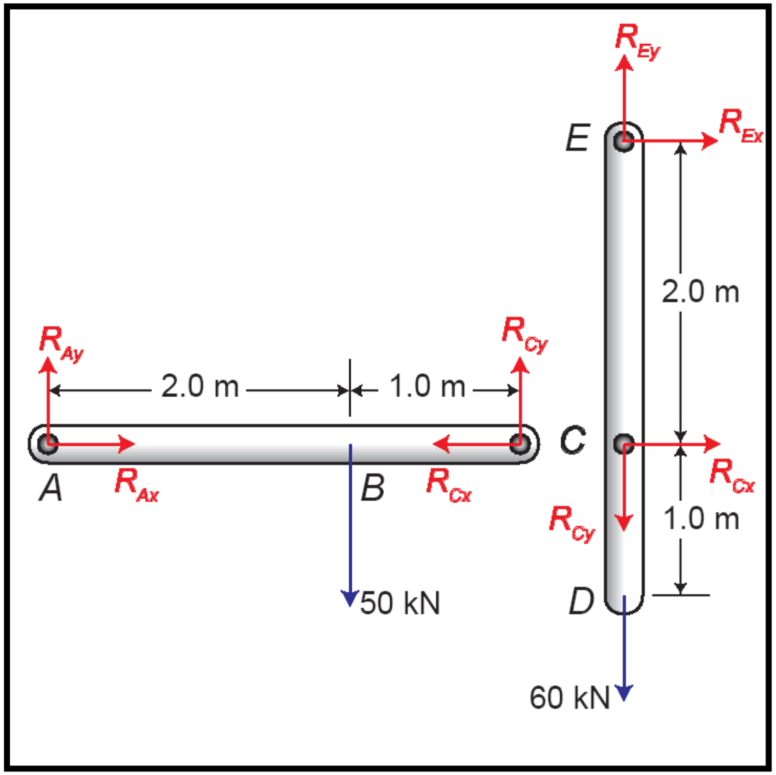
Draw the free body diagrams for *DCE* and for *ABC* (notice that, since *C* is in equilibrium, the reactions at *C* have to be equal and opposite):



Example:

Take moments about *E* to find R_{Cx}:

$$\Sigma M_E = R_{Cx} \cdot (2.0) = 0$$
$$R_{Cx} = 0$$



Take moments about *A* to find R_{Cy}:

$$\Sigma M_A = R_{Cy} \cdot (3.0) - (50.0) \cdot (2.0) = 0$$
$$R_{Cy} = \frac{2.0}{3.0} \cdot (50.0)$$
$$= 33.333 \text{ kN}$$

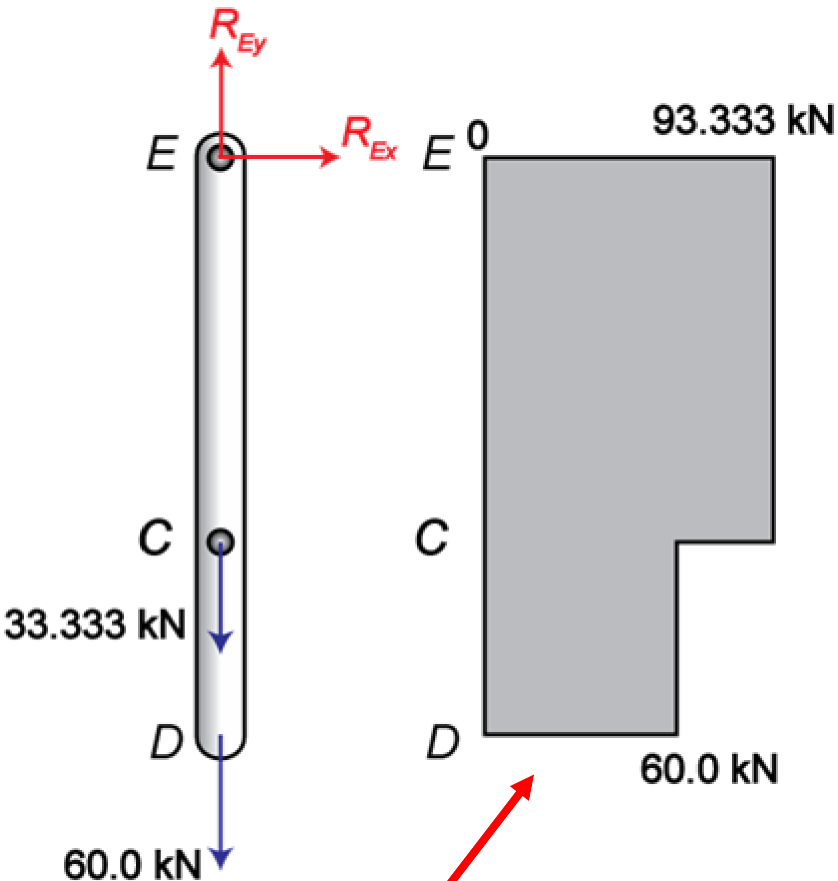
Example:

Now examine the forces in *DCE*:

$$\Sigma F_x = R_{Ex} \cdot (2.0) = 0$$
$$R_{Ex} = 0$$

Since $R_{Ex} = 0$, all the forces are in the *y* – direction (along the axis of *DCE*). The internal axial forces are:

$$\Sigma T_{DC} = 60.0 \text{ kN}$$
$$\Sigma T_{CE} = 93.333 \text{ kN}$$



The internal axial force diagram is as shown