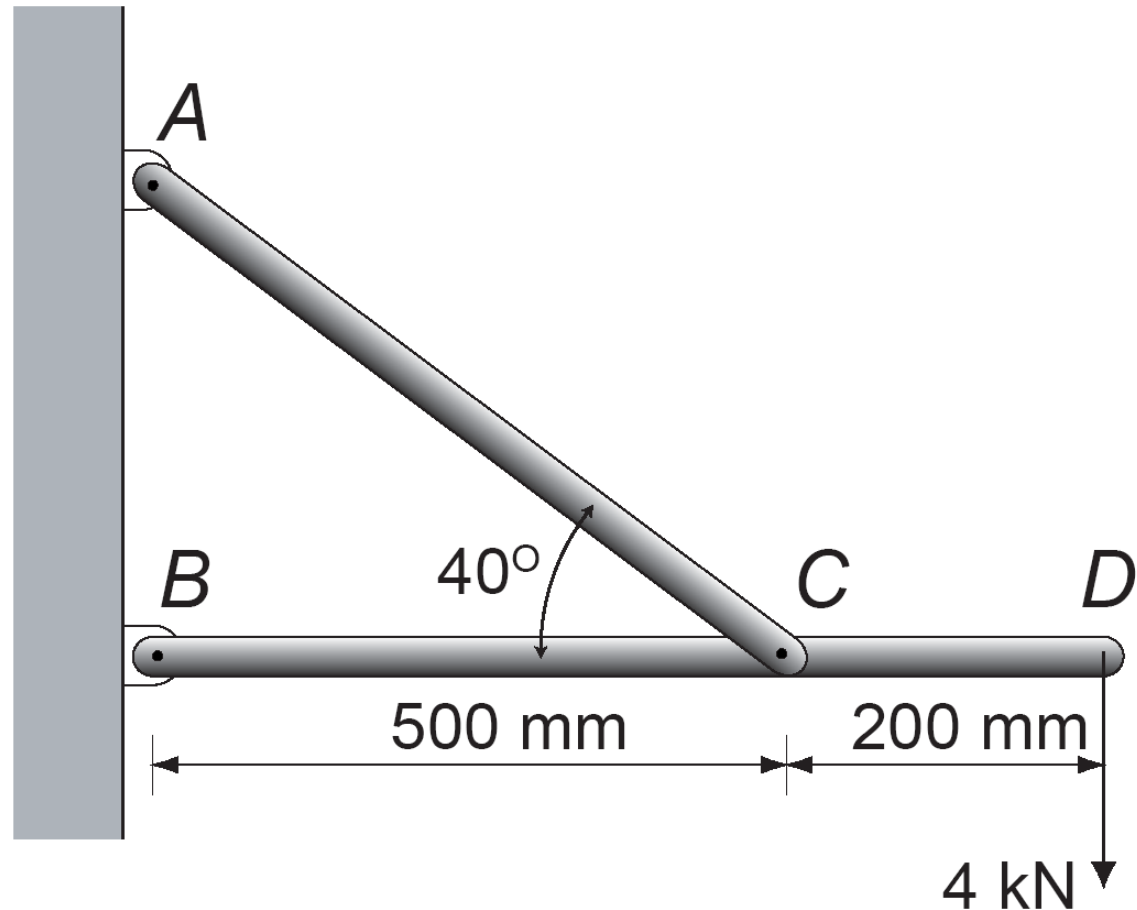


# Stress

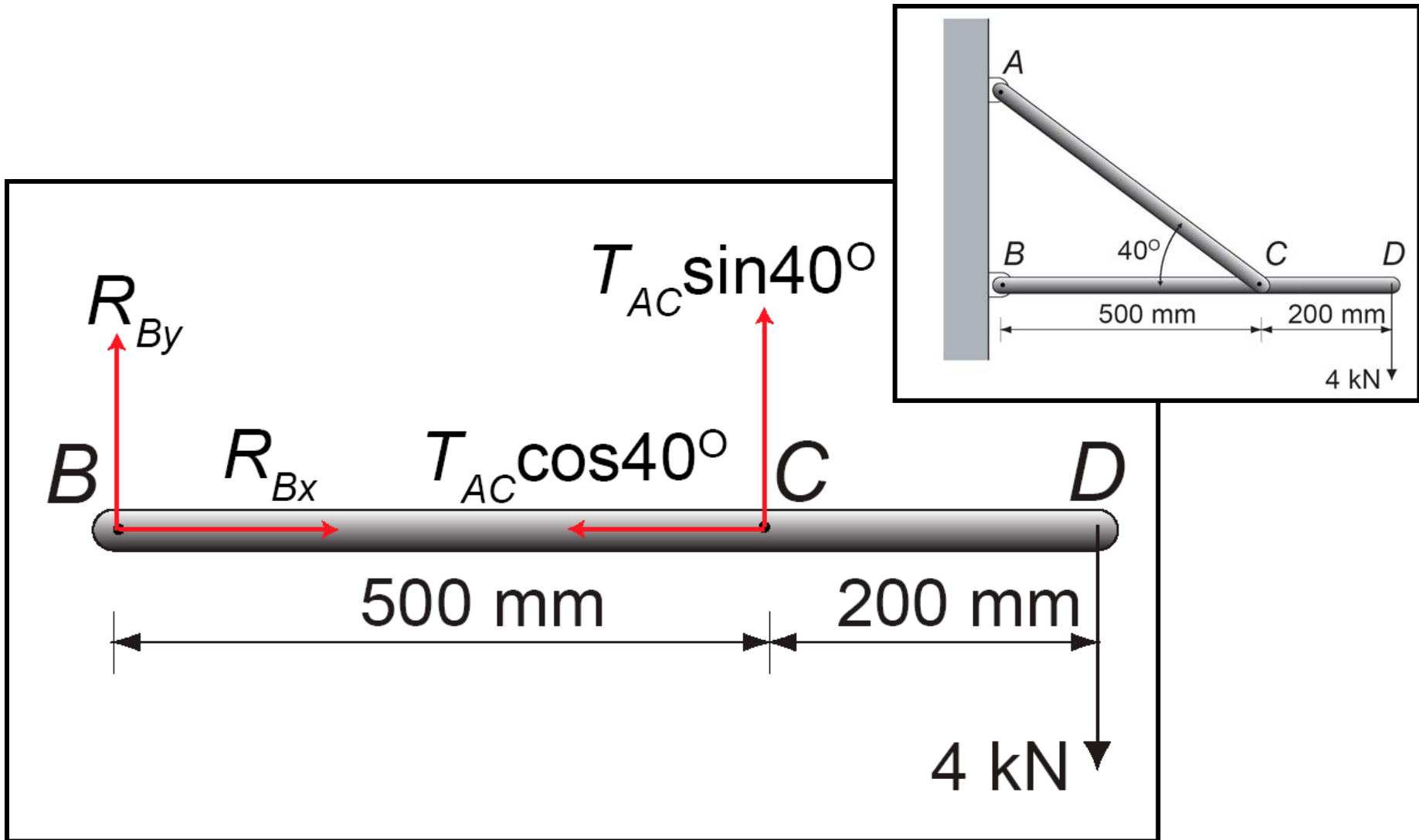
(Shear and Bearing Stress)

Strength of Materials

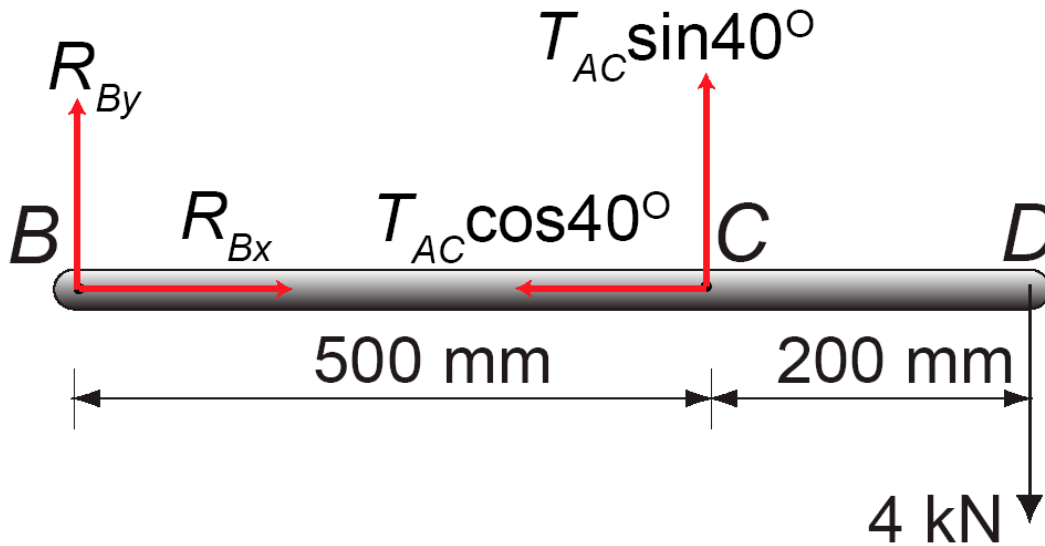
Example:



Bolts at  $A$  and  $B$  are in single shear with an allowable shear stress of  $\tau_{\text{allow}} = 120 \text{ MPa}$ . Select the minimum size of bolt required if bolts are available in diameters with one mm increments

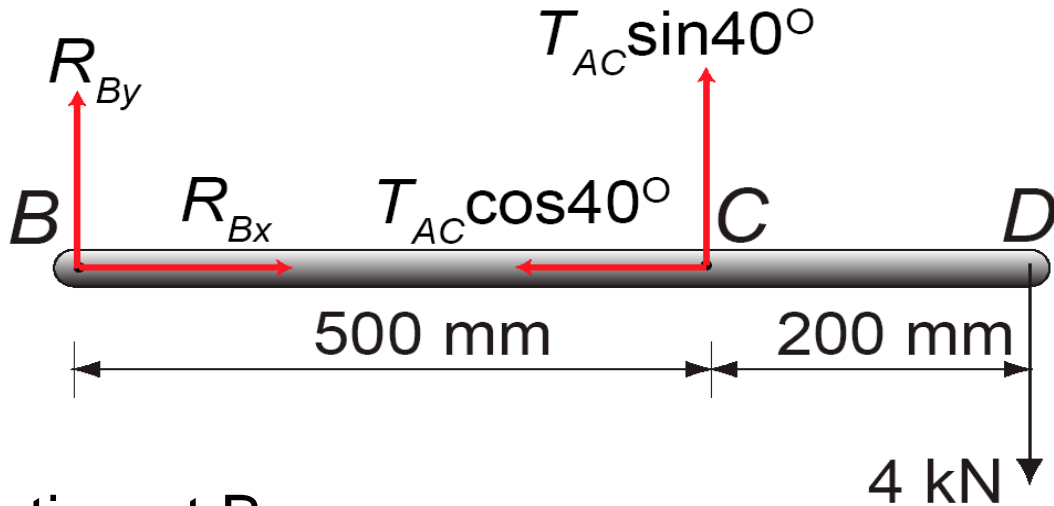


Draw the free body diagram of BCD



Take moments about  $B$  to find the tension in  $AC$ :

$$\begin{aligned} \Sigma M_B &= T_{AC} \sin 40^\circ \times 500 - 4 \times 700 = 0 \text{ kN} \cdot \text{mm} \\ T_{AC} &= \frac{4 \times 700}{\sin 40^\circ \times 500} \\ &= 8.7121 \text{ kN} \\ &= 8712.1 \text{ N} \end{aligned}$$



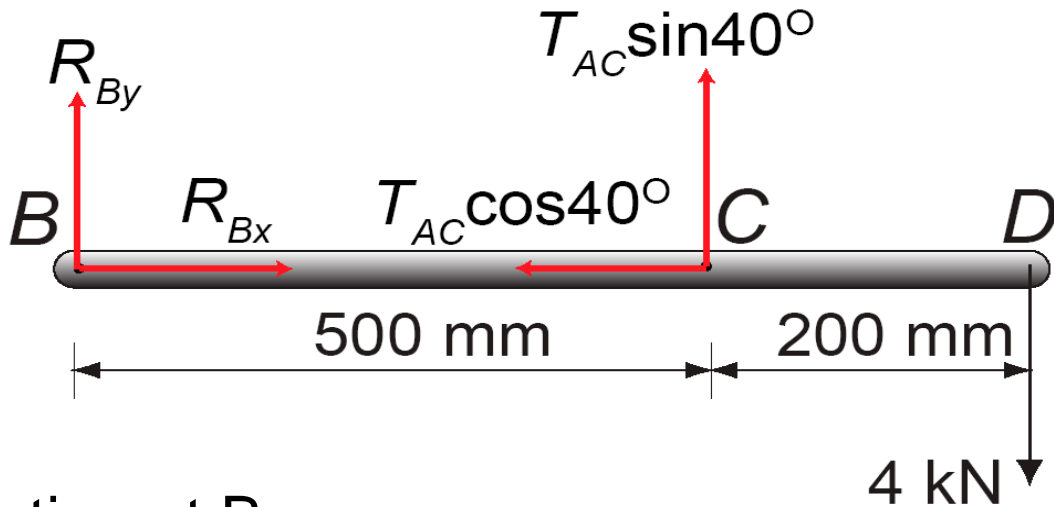
Find the reaction at B:

$$\Sigma F_y = R_{By} + T_{AC} \sin 40^\circ - 4.0 = 0$$

$$\begin{aligned} R_{By} &= 4.0 - T_{AC} \sin 40^\circ \\ &= -1600.0 \text{ N} \end{aligned}$$

$$\Sigma F_x = R_{Bx} - T_{AC} \cos 40^\circ = 0$$

$$\begin{aligned} R_{Bx} &= T_{AC} \cos 40^\circ \\ &= 6673.9 \text{ N} \end{aligned}$$



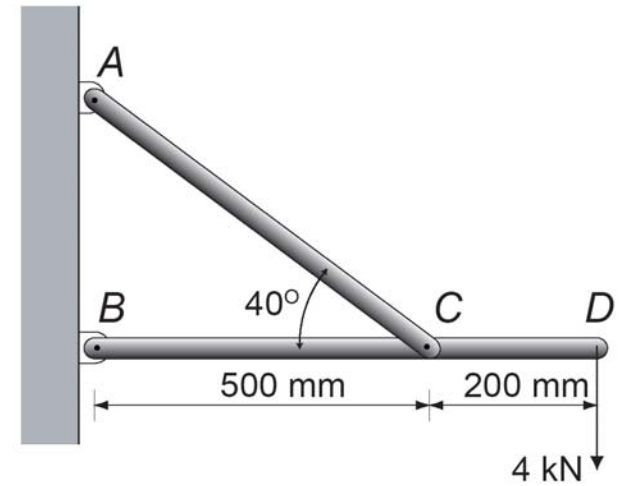
Find the reaction at B:

$$\begin{aligned}
 R_B &= \sqrt{R_{Bx}^2 + R_{By}^2} \\
 &= \sqrt{(6673.9)^2 + (-1600.0)^2} \\
 &= 6863.0 \text{ N}
 \end{aligned}$$

We don't need the direction of the reaction at B, just the magnitude.

Let the diameter of the bolt at A be  $d_A$ . The tension in AC is equal to the internal resisting force:

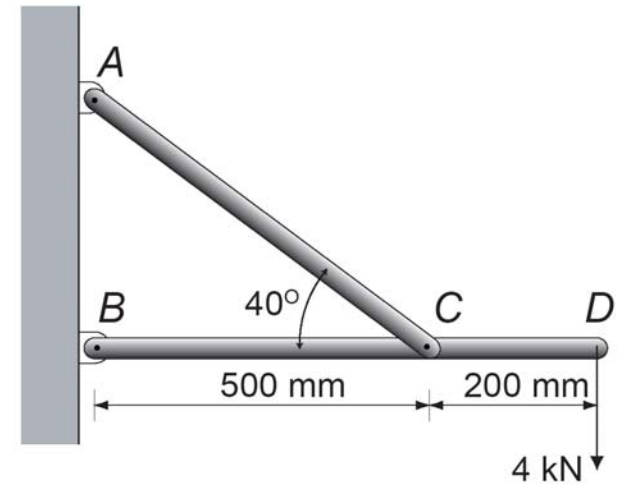
$$\begin{aligned}\tau_{allow} &= P/A_S \\ &= \frac{T_{AC}}{\pi (d_A)^2 / 4} \\ d_A^2 &= \frac{4T_{AC}}{\pi \cdot \tau_{allow}} \\ &= \frac{4 \times 8712.1 \text{ N}}{\pi \times 120 \text{ N/mm}^2} \\ &= 92.438 \text{ mm}^2 \\ d_A &= 9.6145 \text{ mm}\end{aligned}$$



A 10 mm diameter bolt is required at A

Let the diameter of the bolt at  $B$  be  $d_B$ . The reaction at  $B$  is equal to the internal resisting force on the bolt:

$$\begin{aligned}
 \tau_{allow} &= P/A_S \\
 &= \frac{R_B}{\pi (d_B)^2 / 4} \\
 d_B^2 &= \frac{4R_B}{\pi \cdot \tau_{allow}} \\
 &= \frac{4 \times 6863.0 \text{ N}}{\pi \times 120 \text{ N/mm}^2} \\
 &= 72.819 \text{ mm}^2 \\
 d_B &= 8.5334 \text{ mm}
 \end{aligned}$$

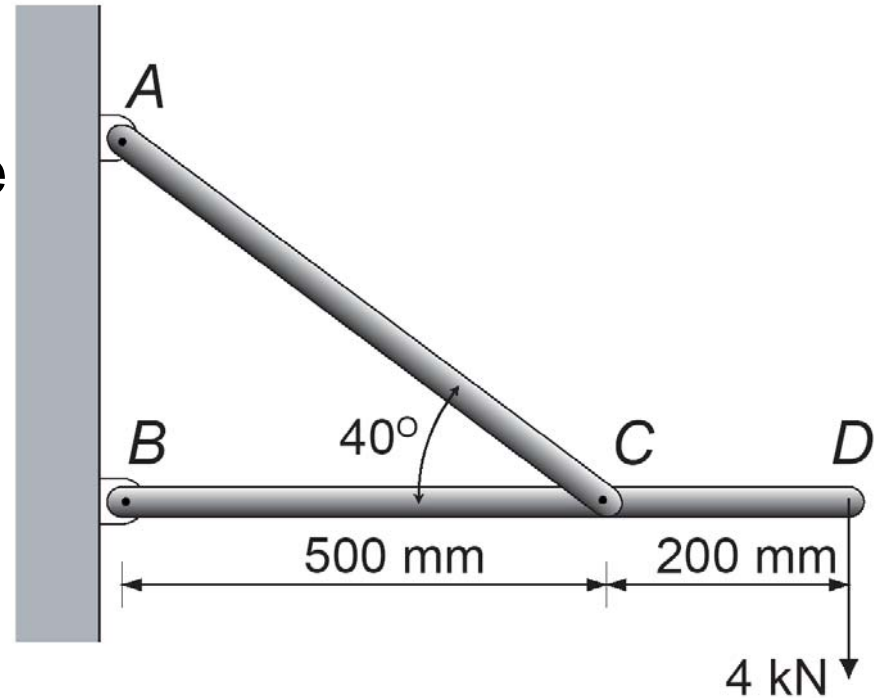


A 9 mm diameter bolt is required at  $B$



## Exercise:

Repeat the previous exercise but assume that the bolts are now in **double** shear.



# Solutions:

$$\begin{aligned}\sigma_{allow} &= \frac{P/2A_S}{T_{AC}} \\ &= \frac{T_{AC}}{2\pi (d_A)^2 / 4} \\ d_A^2 &= \frac{4T_{AC}}{2\pi\sigma_{allow}} \\ &= \frac{4 \times 8712.1 \text{ N}}{2\pi \times 120 \text{ N/mm}^2} \\ &= 46.219 \text{ mm}^2 \\ d_A &= 6.7985 \text{ mm}\end{aligned}$$

$$\begin{aligned}\sigma_{allow} &= \frac{P/2A_S}{R_B} \\ &= \frac{R_B}{2\pi (d_B)^2 / 4} \\ d_B^2 &= \frac{4R_B}{2\pi\sigma_{allow}} \\ &= \frac{4 \times 6863.0 \text{ N}}{2\pi \times 120 \text{ N/mm}^2} \\ &= 36.409 \text{ mm}^2 \\ d_B &= 6.0340 \text{ mm}\end{aligned}$$

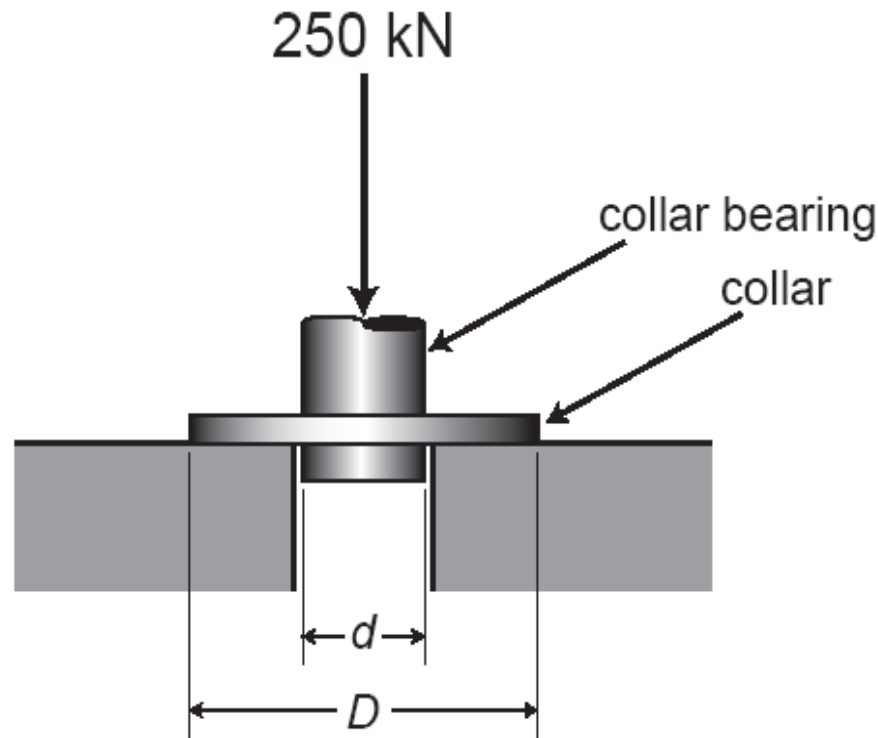
7 mm bolt required at A and 7 mm bolt required at B

# Bearing Stress

- when one body presses against another, **bearing stress** occurs
- this is what we typically think of as pressure
- bearing stress is assumed to be uniformly distributed over the contact area
- bearing stress is given by:

$$\sigma_b = \frac{P}{A_b}$$

## Example:



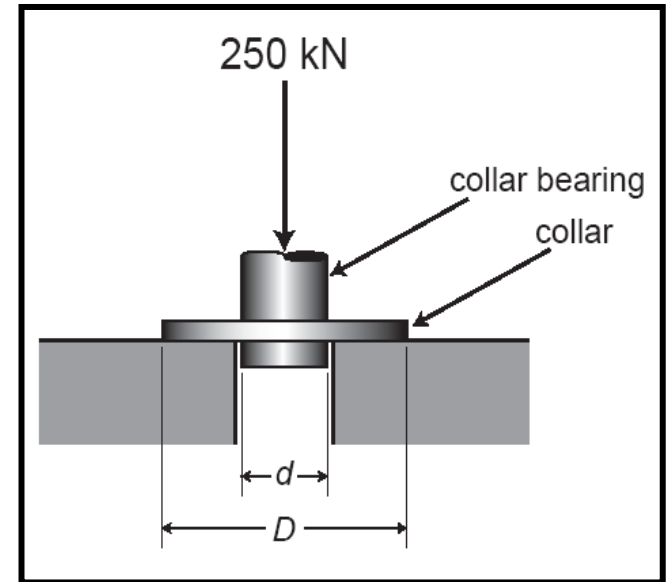
The collar bearing shown is subject to a 250 kN force. The collar is 12.5 mm thick.

Find  $d$  and  $D$  (to the nearest mm) so that the allowable normal stress in the column of  $\sigma = 140$  Mpa, the allowable shear stress in the collar of  $\tau = 100$  Mpa and the allowable bearing stress between the collar and the support of  $\sigma_b = 35$  Mpa is not exceeded.

## Solution:

Find  $d$  so that the normal stress in the column is not exceeded:

$$\begin{aligned}\sigma = P/A &= \frac{P}{\pi d^2/4} \\ d &= \sqrt{\frac{4P}{\sigma\pi}} \\ &= \sqrt{\frac{4 \times 250000 \text{ N}}{140\pi \text{ N/mm}^2}} \\ &= 47.683 \text{ mm}\end{aligned}$$

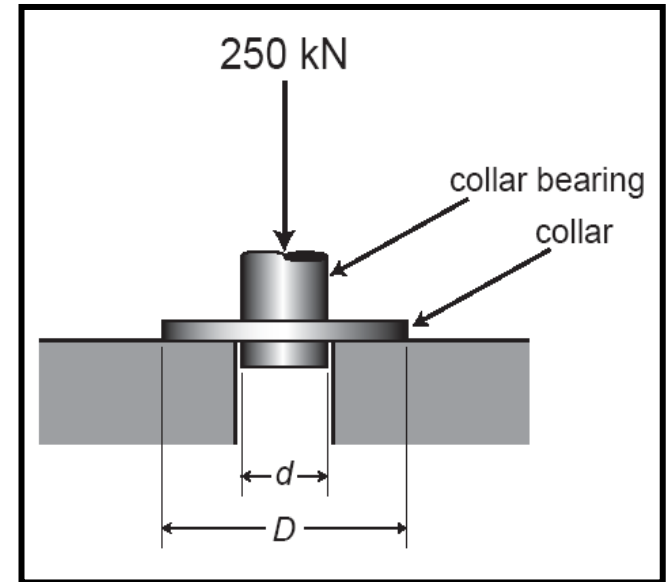


A diameter of  $d = 48 \text{ mm}$  is required for the column so that the allowable normal stress is not exceeded

## Solution:

Find  $d$  so that the shear stress in the collar is not exceeded (note:  $t$  is the thickness of the collar):

$$\begin{aligned}\tau = P/A_s &= \frac{P}{\pi dt} \\ d &= \frac{P}{\pi \tau t} \\ &= \frac{250000 \text{ N}}{\pi \times 100 \text{ N/mm}^2 \times 12.5 \text{ mm}} \\ &= 63.662 \text{ mm}\end{aligned}$$



A diameter of 64 mm is required for the column so that the shear stress is not exceeded

Choose  $d = 64 \text{ mm}$  so that both  $\sigma_{\text{allow}}$  and  $\tau_{\text{allow}}$  are satisfied

Find  $D$  so that the bearing stress is not exceeded:

$$\sigma_b = P/A_b = \frac{P}{\pi (D^2 - d^2) / 4}$$

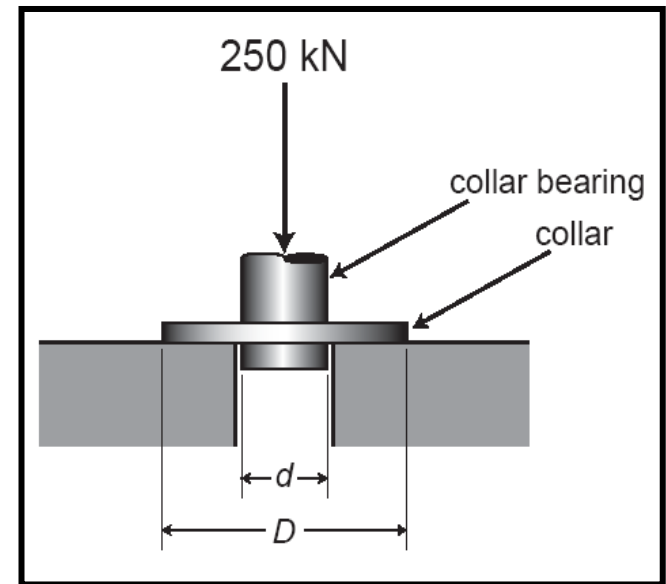
$$D^2 - d^2 = \frac{4P}{\pi \sigma_b}$$

$$D^2 = \frac{4P}{\pi \sigma_b} + d^2$$

$$D = \sqrt{\frac{4P}{\pi \sigma_b} + d^2}$$

$$= \sqrt{\frac{4 \times 250000 \text{ N}}{\pi \times 35 \text{ N/mm}^2} + (64 \text{ mm})^2}$$

$$= 114.85 \text{ mm}$$



A diameter of  $D = 115 \text{ mm}$  is required for the collar so that the allowable bearing stress is not exceeded