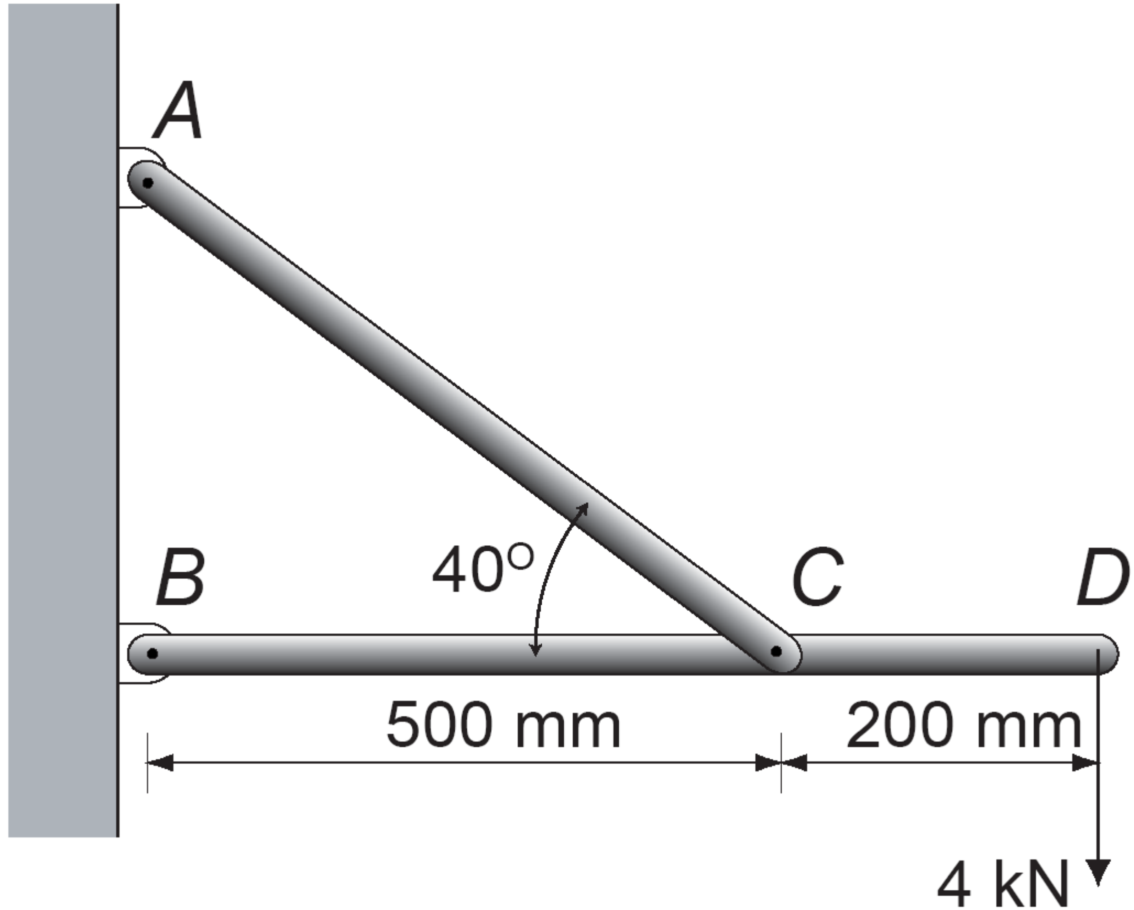
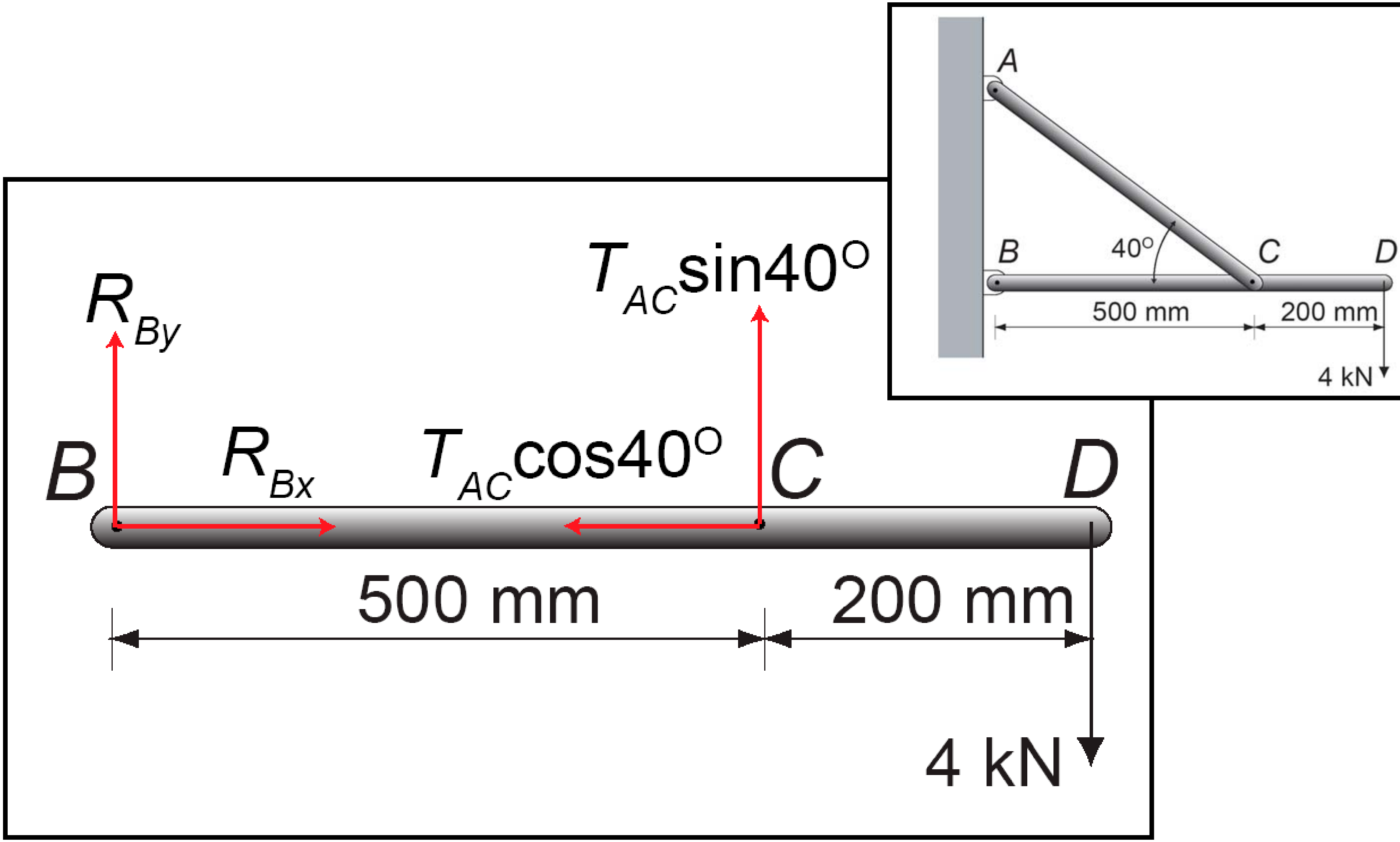
Stress
(Shear and Bearing Stress)

Strength of Materials

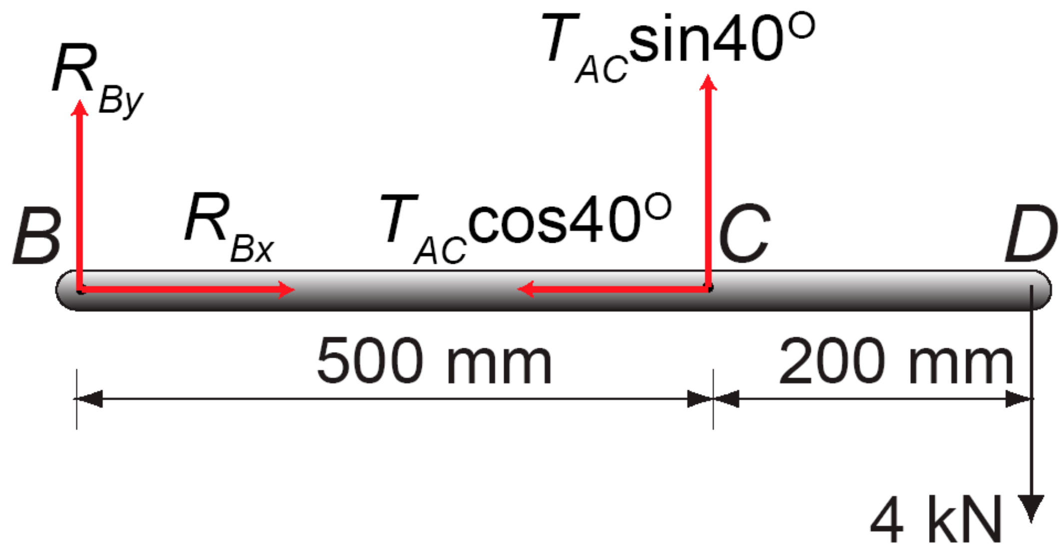
<u>Example</u>:



Bolts at *A* and *B* are in single shear with an allowable shear stress of τ_{allow} = 120 MPa. Select the minimum size of bolt required if bolts are available in diameters with one mm increments



Draw the free body diagram of BCD



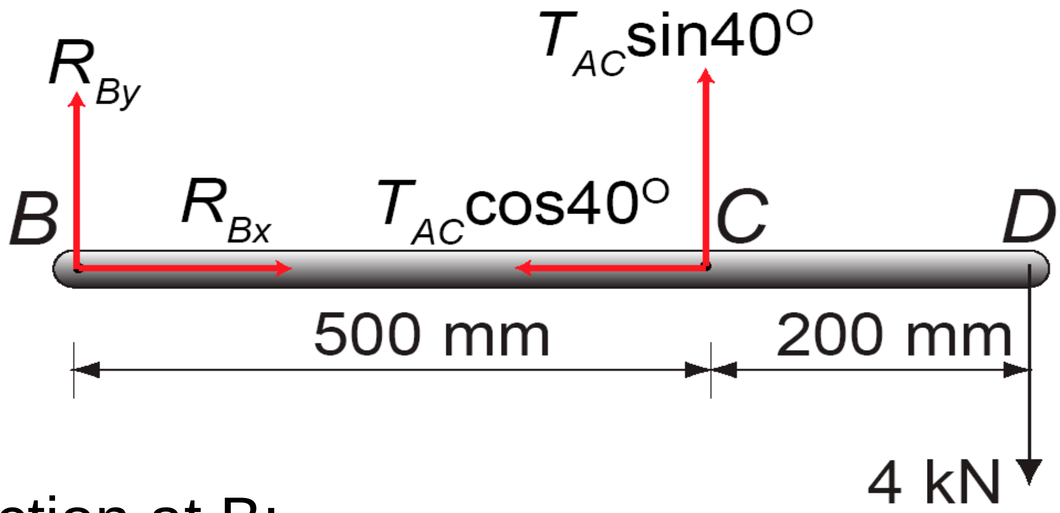
Take moments about *B* to find the tension in *AC:*

$$\Sigma M_B = T_{AC}\sin 40^\circ \times 500 - 4 \times 700 = 0 \text{ kN} \cdot \text{mm}$$

$$T_AC = \frac{4 \times 700}{\sin 40^\circ \times 500}$$

$$= 8.7121 \text{ kN}$$

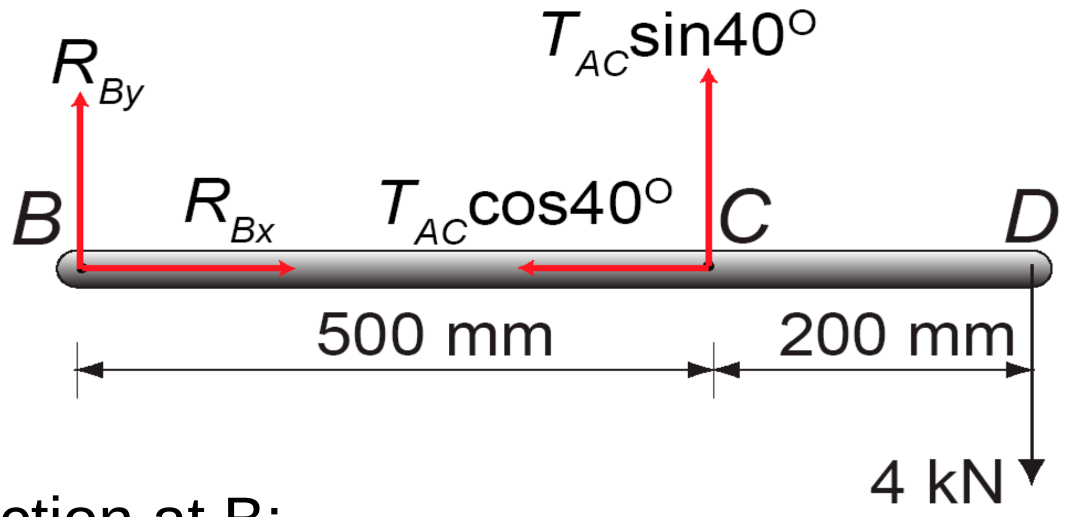
$$= 8712.1 \text{ N}$$



Find the reaction at B:

$$
\begin{aligned}
\Sigma F_y = R_{By} + T_{AC}\,\mathrm{sin}\,40^\circ - 4.0 &= 0 \\
R_{By} &= 4.0 - T_{AC}\,\mathrm{sin}\,40^\circ \\
&= -1600.0\ \mathrm{N}
\end{aligned}
$$

$$
\begin{aligned}
\Sigma F_x = R_{Bx} - T_{AC}\,\mathrm{cos}\,40^\circ &= 0 \\
R_{Bx} &= T_{AC}\,\mathrm{cos}\,40^\circ \\
&= 6673.9\ \mathrm{N}
\end{aligned}
$$

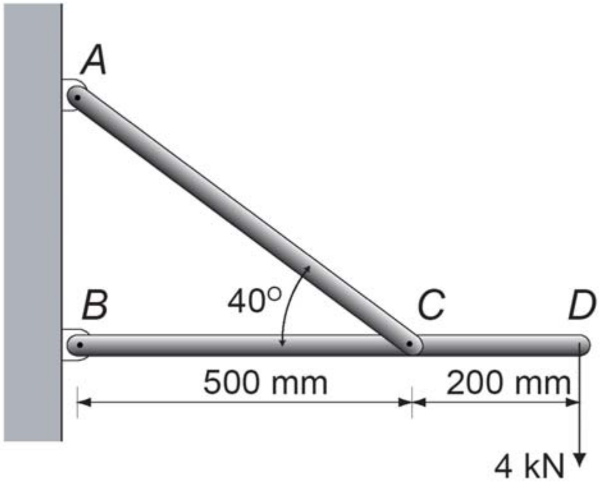


Find the reaction at B:

$$
\begin{aligned}
R_B &= \sqrt{R_{Bx}^2 + R_{By}^2} \\
&= \sqrt{(6673.9)^2 + (-1600.0)^2} \\
&= 6863.0 \text{ N}
\end{aligned}
$$

We don’t need the direction of the reaction at B, just the magnitude.

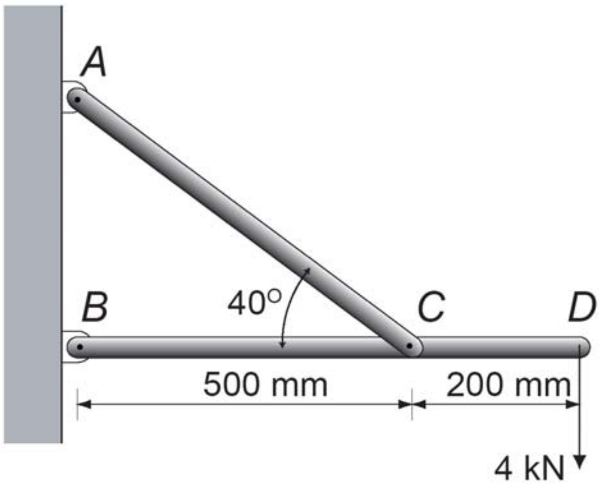
Let the diameter of the bolt at *A* be d_A. The tension in *AC* is equal to the internal resisting force:



$$
\begin{aligned}
\tau_{allow} &= P/A_S \\
&= \frac{T_{AC}}{\pi (d_A)^2/4} \\
d_A^2 &= \frac{4T_{AC}}{\pi \cdot \tau_{allow}} \\
&= \frac{4 \times 8712.1 \text{ N}}{\pi \times 120 \text{ N/mm}^2} \\
&= 92.438 \text{ mm}^2 \\
d_A &= 9.6145 \text{ mm}
\end{aligned}
$$

A 10 mm diameter bolt is required at *A*

Let the diameter of the bolt at *B* be d_B. The reaction at *B* is equal to the internal resisting force on the bolt:



$$\tau_{allow} = P/A_S = \frac{R_B}{\pi (d_B)^2/4}$$

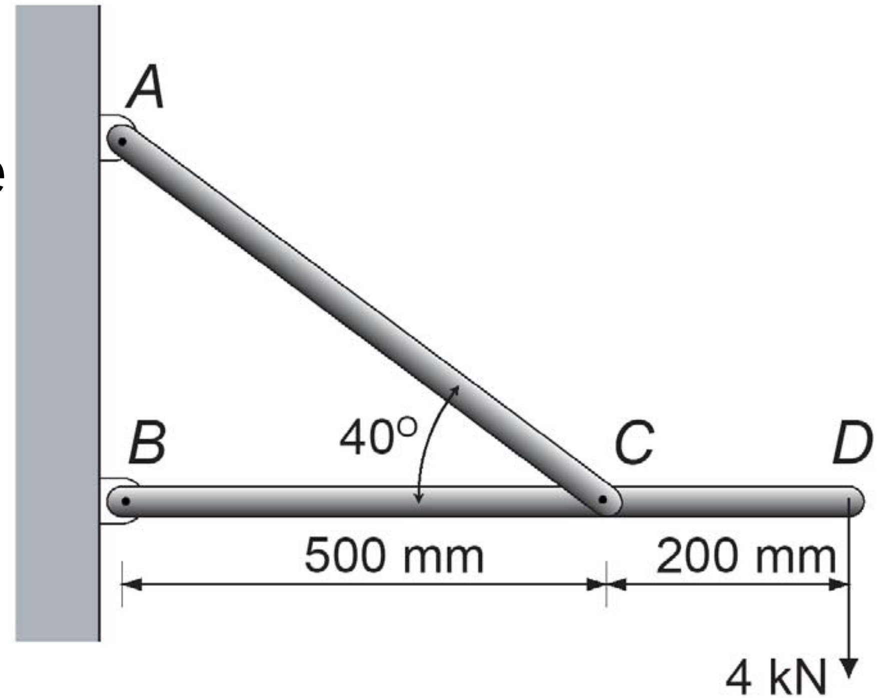
$$d_B^2 = \frac{4R_B}{\pi \cdot \tau_{allow}} = \frac{4 \times 6863.0 \text{ N}}{\pi \times 120 \text{ N/mm}^2} = 72.819 \text{ mm}^2$$

$$d_B = 8.5334 \text{ mm}$$

A 9 mm diameter bolt is required at *B*

Exercise:

Repeat the previous exercise but assume that the bolts are now in **double** shear.



Solutions:

$$\begin{aligned}
\sigma_{allow} &= P/2A_S \\
&= \frac{T_{AC}}{2\pi\,(d_A)^2/4} \\
d_A^2 &= \frac{4T_{AC}}{2\pi\sigma_{allow}} \\
&= \frac{4 \times 8712.1 \text{ N}}{2\pi \times 120 \text{ N/mm}^2} \\
&= 46.219 \text{ mm}^2 \\
d_A &= 6.7985 \text{ mm}
\end{aligned}$$

$$\begin{aligned}
\sigma_{allow} &= P/2A_S \\
&= \frac{R_B}{2\pi\,(d_B)^2/4} \\
d_B^2 &= \frac{4R_B}{2\pi\sigma_{allow}} \\
&= \frac{4 \times 6863.0 \text{ N}}{2\pi \times 120 \text{ N/mm}^2} \\
&= 36.409 \text{ mm}^2 \\
d_B &= 6.0340 \text{ mm}
\end{aligned}$$

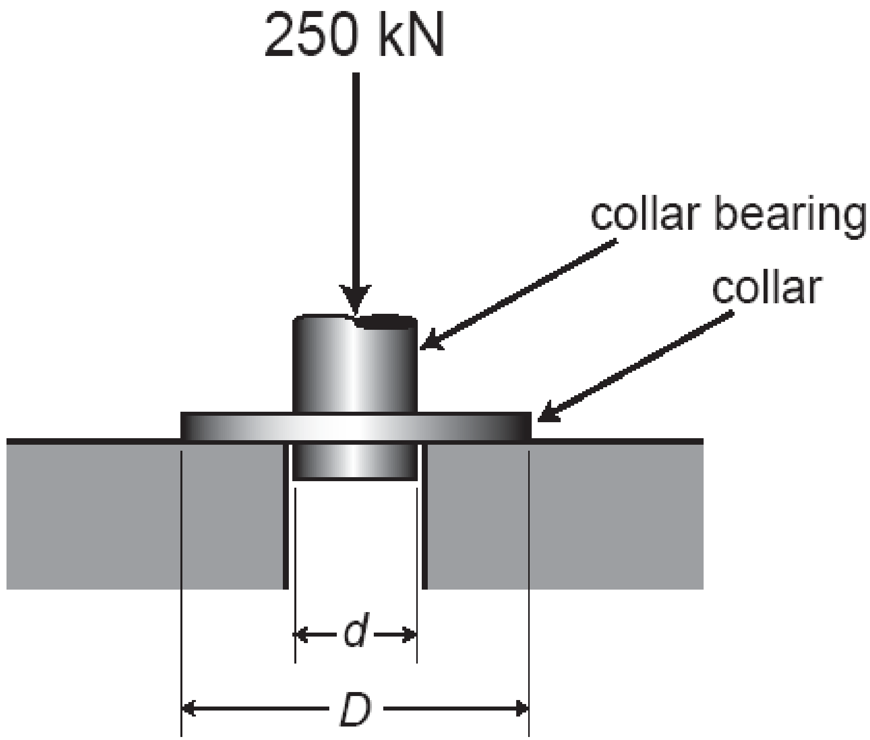
7 mm bolt required at *A* and 7 mm bolt required at *B*

Bearing Stress

- when one body presses against another, **bearing stress** occurs
- this is what we typically think of as pressure
- bearing stress is assumed to be uniformly distributed over the contact area
- bearing stress is given by:

$$\sigma_b = \frac{P}{A_b}$$

Example:

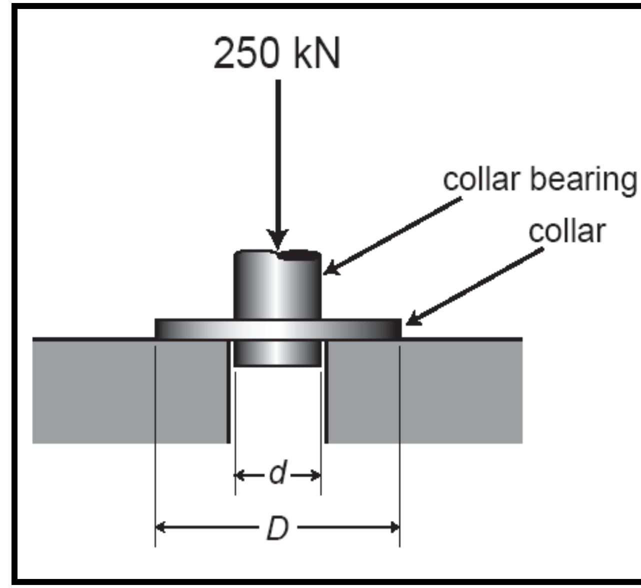


The collar bearing shown is subject to a 250 kN force. The collar is 12.5 mm thick.

Find *d* and *D* (to the nearest mm) so that the allowable normal stress in the column of σ = 140 Mpa, the allowable shear stress in the collar of τ = 100 Mpa and the allowable bearing stress between the collar and the support of σ_b = 35 Mpa is not exceeded.

Solution:

Find *d* so that the normal stress in the column is not exceeded:



$$\sigma = P/A = \frac{P}{\pi d^2/4}$$

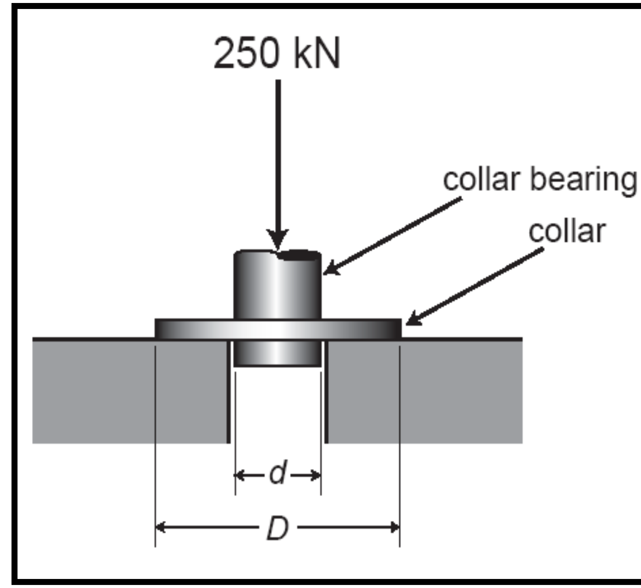
$$d = \sqrt{\frac{4P}{\sigma\pi}}$$

$$= \sqrt{\frac{4 \times 250000 \text{ N}}{140\pi \text{ N/mm}^2}}$$

$$= 47.683 \text{ mm}$$

A diameter of *d* = 48 mm is required for the column so that the allowable normal stress is not exceeded

Solution:



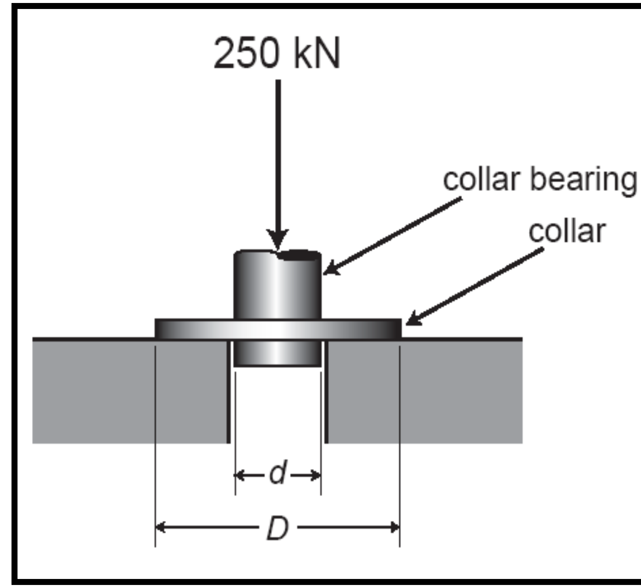
Find *d* so that the shear stress in the collar is not exceeded (note: *t* is the thickness of the collar):

$$\begin{aligned} \tau = P/A_s &= \frac{P}{\pi d t} \\ d &= \frac{P}{\pi \tau t} \\ &= \frac{250000 \text{ N}}{\pi \times 100 \text{ N/mm}^2 \times 12.5 \text{ mm}} \\ &= 63.662 \text{ mm} \end{aligned}$$

A diameter of 64 mm is required for the column so that the shear stress is not exceeded

Choose *d* = 64 mm so that both σ_{allow} and τ_{allow} are satisfied

Find *D* so that the bearing stress is not exceeded:



$$\sigma_b = P/A_b = \frac{P}{\pi\left(D^2 - d^2\right)/4}$$

$$D^2 - d^2 = \frac{4P}{\pi\sigma_b}$$

$$D^2 = \frac{4P}{\pi\sigma_b} + d^2$$

$$D = \sqrt{\frac{4P}{\pi\sigma_b} + d^2}$$

$$= \sqrt{\frac{4 \times 250000\ \text{N}}{\pi \times 35\ \text{N/mm}^2} + (64\ \text{mm})^2}$$

$$= 114.85\ \text{mm}$$

A diameter of *D* = 115 mm is required for the collar so that the allowable bearing stress is not exceeded