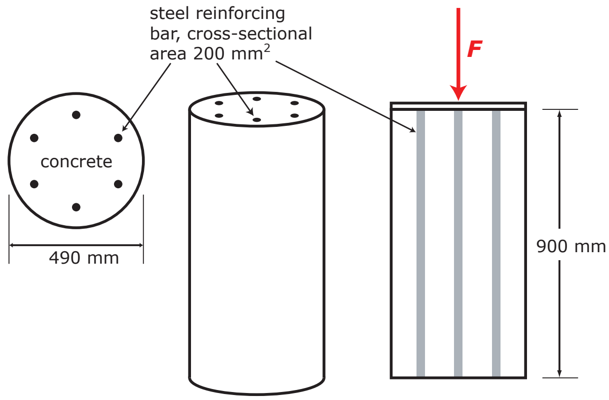
Assignment 3 - Question 3 Solution

Question 3: A concrete column, length $L = 900$ mm and diameter $d = 490$ mm, is reinforced with 6 steel rods (rebar), each having a cross-sectional area of 200 mm^2. The Young's Moduli are $E_S = 200$ GPa and $E_C = 27$ GPa . The allowable stresses for each material are $\sigma_{allow_S} = 125$ MPa and $\sigma_{allow_C} = 29.2$ MPa .



(1) Find the maximum allowable load, F (3312 kN)
(2) Find the stress in the concrete with this load (16.876 MPa) - this answer originally posted incorrectly as 16.882 :-(
(3) Find the stress in the steel with this load (125 MPa)
(4) Find the deformation in length with this load (0.5625 mm)

Part (1)

Let P_s be the internal force in each steel rod and let P_C be the internal force in the concrete. Then,

(1) $$\Sigma F_y = P_C + 6P_S = F$$

The deformation in the steel is the same as the deformation in the concrete so we have:

$$\begin{aligned}
\delta_C &= \delta_S \\
\left(\frac{PL}{AE}\right)_C &= \left(\frac{PL}{AE}\right)_S \\
\frac{P_C \times 900}{\left(\frac{\pi(490)^2}{4} - 1200\right) \times 27 \times 10^3} &= \frac{P_S \times 900}{200 \times 200 \times 10^3} \\
P_C &= 126.48 P_S \qquad (2)
\end{aligned}$$

Substituting for P_C into equation (1) gives the following results:

$$\begin{aligned}
126.48P_S + 6P_S &= F \\
132.48P_S &= F \qquad (3)
\end{aligned}$$

and from equation(2),

$$\begin{aligned}
P_C &= \frac{126.48F}{132.48} \\
&= 0.95471F \\
1.0474P_C &= F \qquad (4)
\end{aligned}$$

Find the maximum allowable force in the steel, P_S, subject to $\sigma_{allow_S} = 125$ MPa :

$$\begin{aligned}
\sigma_{allow_S} &= \frac{P_{allow_S}}{A_S} \\
P_{allow_S} &= \sigma_{allow_S} \cdot A_S \\
&= 125 \times 200 \\
&= 25000 \text{ N}
\end{aligned}$$

From equation (3), we see that the maximum load F that the steel can support is:

$$\begin{aligned}
F &= 25000 \times 132.48 \text{ N} \\
&= 3312 \text{ kN}
\end{aligned}$$

We must do a similar calculation for the maximum allowable force in the concrete, P_C, subject to $\sigma_{allow_C} = 29.2$ MPa:

$$\begin{aligned}
P_{allow_C} &= \sigma_{allow_C} \cdot A_C \\
&= 29.2 \times \left(\frac{\pi (490)^2}{4} - 1200 \right) \\
&= 5471.3 \text{ kN}
\end{aligned}$$

Thus, from equation (4), the maximum load F that the concrete can support is:

$$\begin{aligned}
F &= 1.0474 \times 5471.3 \\
&= 5730.8 \text{ kN}
\end{aligned}$$

This is a larger load than the 3312 kN that the steel can support. Therefore, the maximum allowable load, F, is **3312 kN**.

Part (2)

What is the stress in the concrete at this load? From equation (4),

$$\begin{aligned}
P_C &= \frac{F}{1.0474} \\
&= \frac{3312}{1.0474} \\
&= 3162.1 \text{ kN} \\
\sigma_C &= \frac{3162.1 \times 10^3}{\left(\frac{\pi (490)^2}{4} - 1200 \right)} \\
&= 16.876 \text{ MPa}
\end{aligned}$$

Part (3)

We have already shown that the stress in the steel at this load is 125 MPa

Part (4)

$$\begin{aligned}
\delta_S &= \sigma \cdot \frac{L}{E} \\
&= 125 \times \frac{900}{200 \times 10^3} \\
&= .5625 \textbf{ mm}
\end{aligned}$$