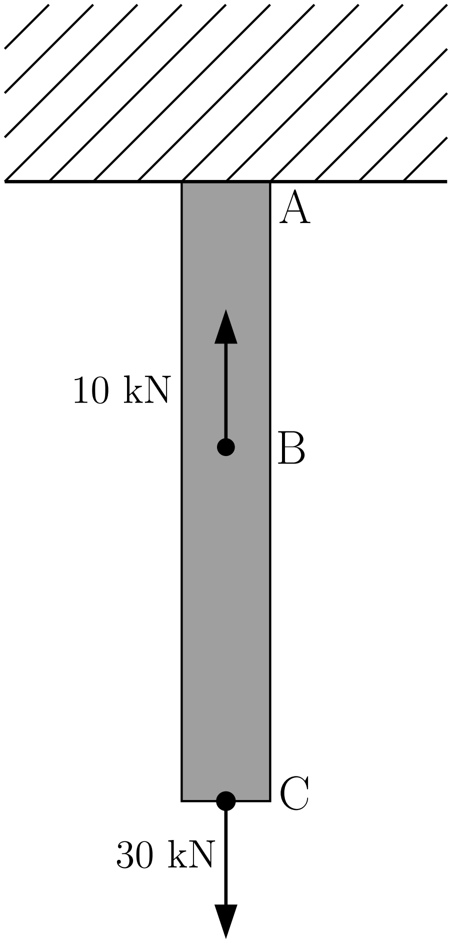
Statically Indeterminate Problems and Problems Involving Two Materials

(Strength of Materials)

Dave Morgan

<dave.morgan@sait.ca>



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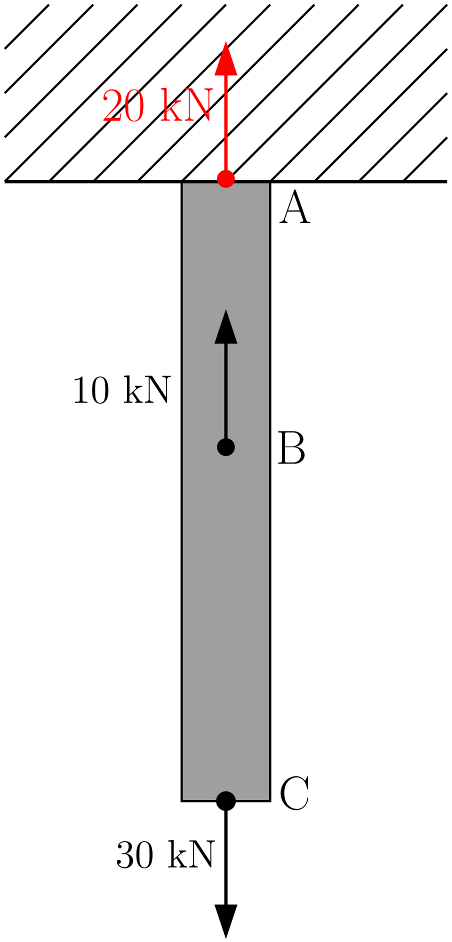

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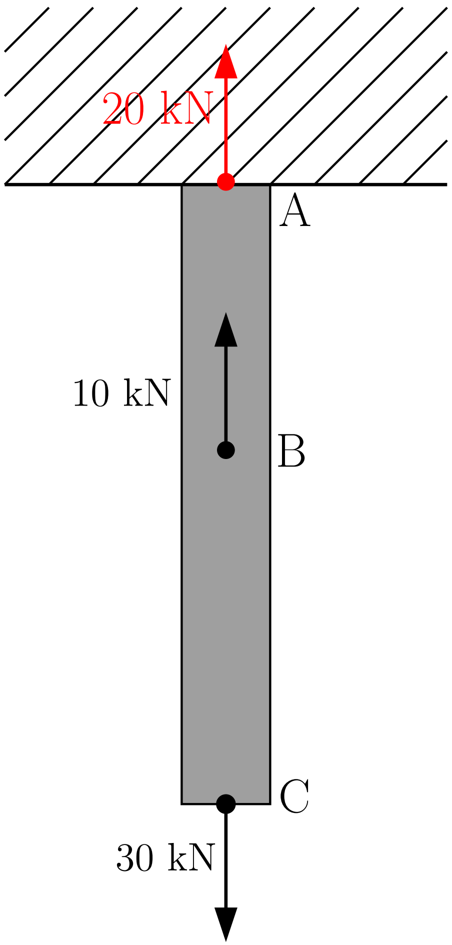
- $\Sigma F_y = 0$, so there is a reaction force of 20 kN at A



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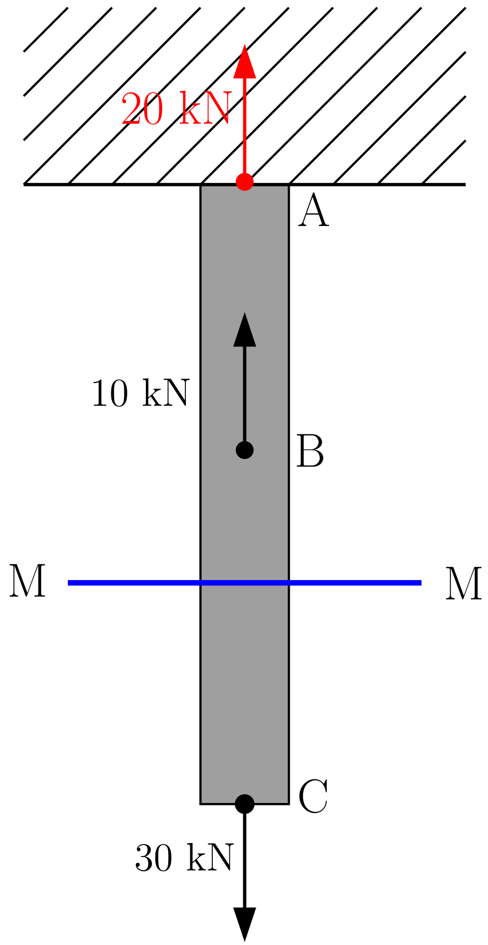

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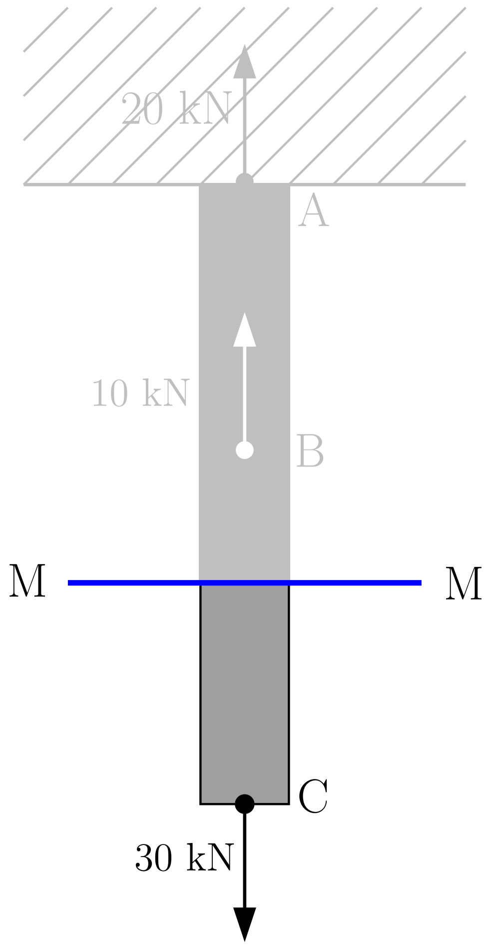

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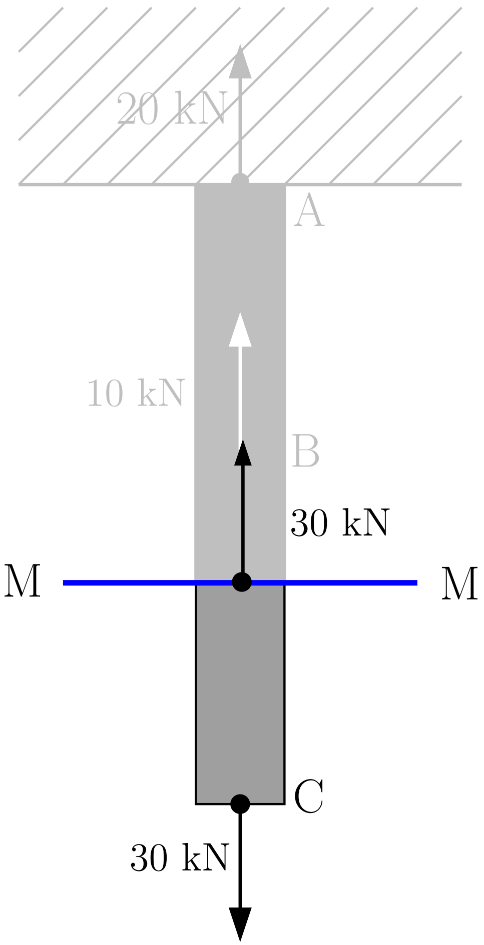
  - Consider only the segment from C to M-M



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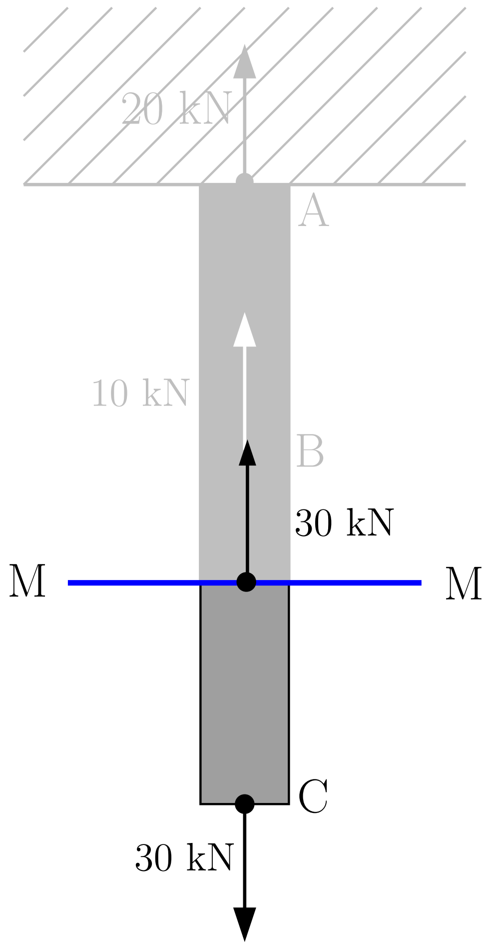
  - $\Sigma F_y = 0$, so there is an internal force of 30 kN at M



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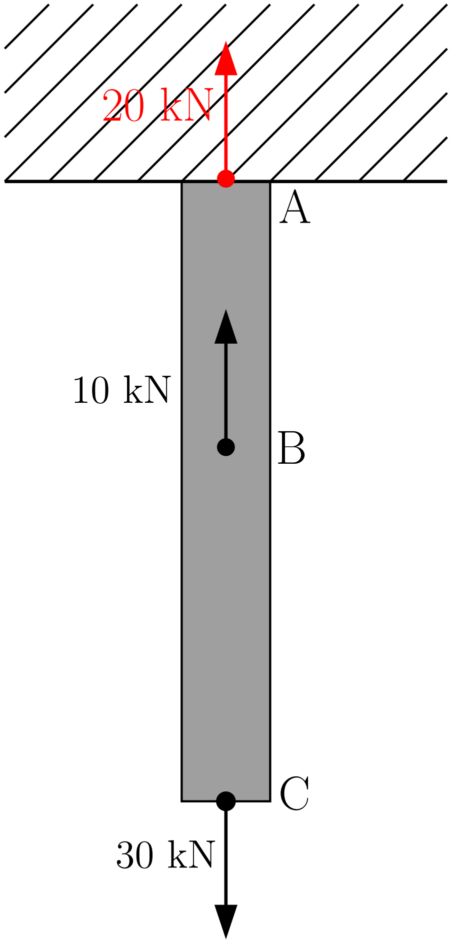
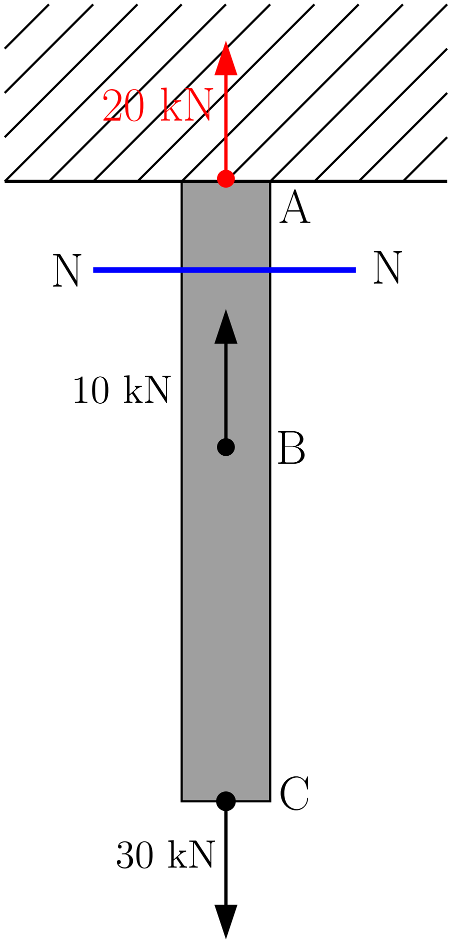
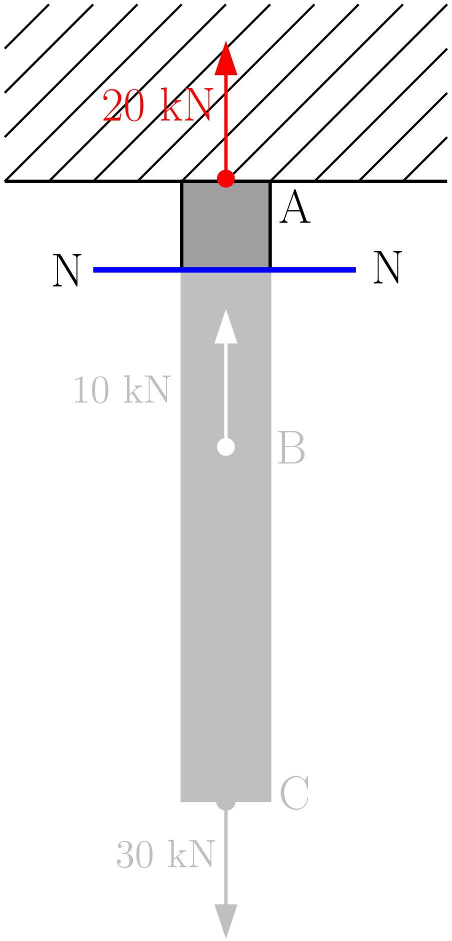
  - $T_{BC} = 30$ kN (tension)



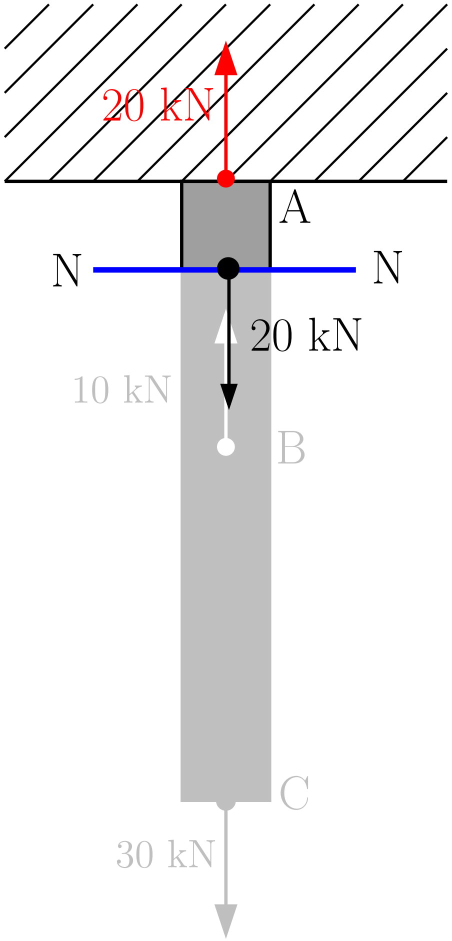
- Similarly, we can find the internal force within segment AB:



- Similarly, we can find the internal force within segment AB:
  - Insert a section N-N through segment AB



- Similarly, we can find the internal force within segment AB:
    - Insert a section N-N through segment AB
    - Consider only the segment from A to N-N



- Similarly, we can find the internal force within segment AB:
  - Insert a section N-N through segment AB
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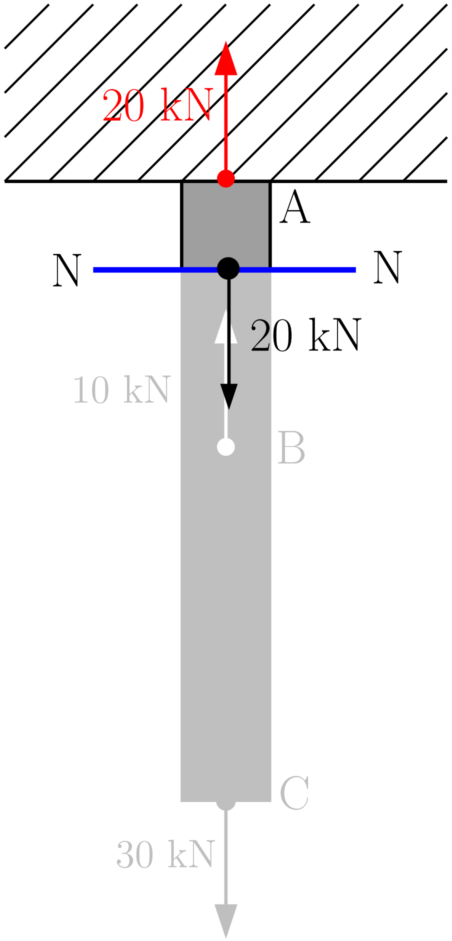

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Structures where forces can be determined using the static equilibrium equations alone ($\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M_A = 0$) are called ***statically determinate*** structures. The previous example is a statically determinate structure.

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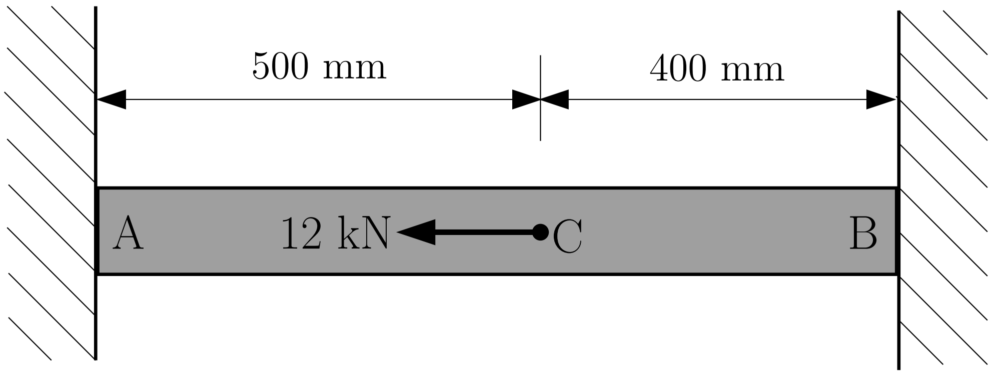
Statically indeterminate structures are often analysed using the conditions of axial deformation given by

$$\delta = \frac{PL}{AE}$$

Example: Consider a bar AB supported at both ends by fixed supports, with an axial force of 12 kN applied at C as illustrated. Find the reactions at the walls



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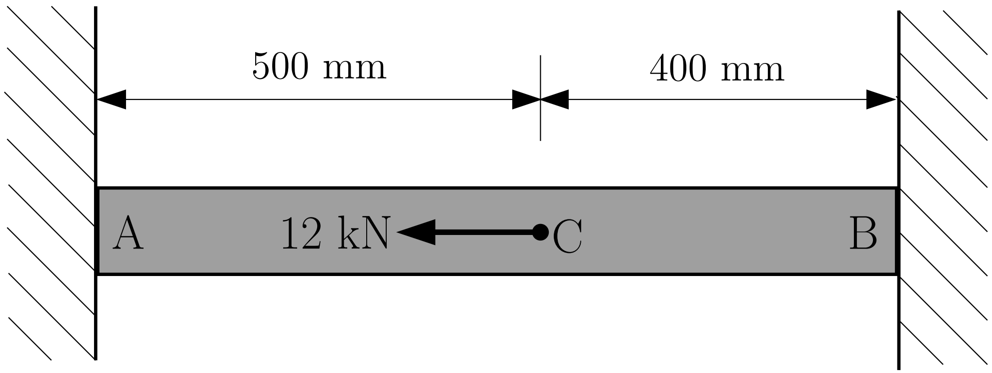

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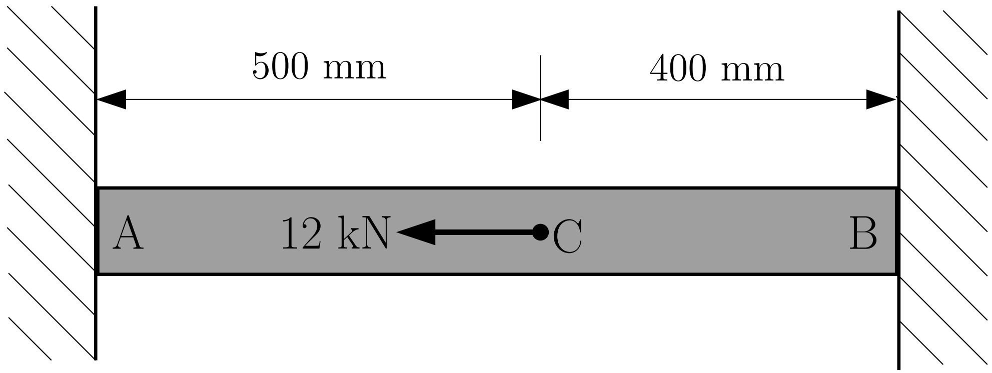

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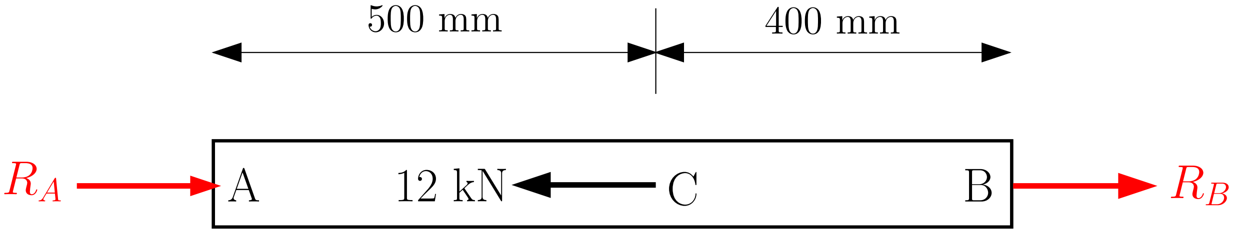
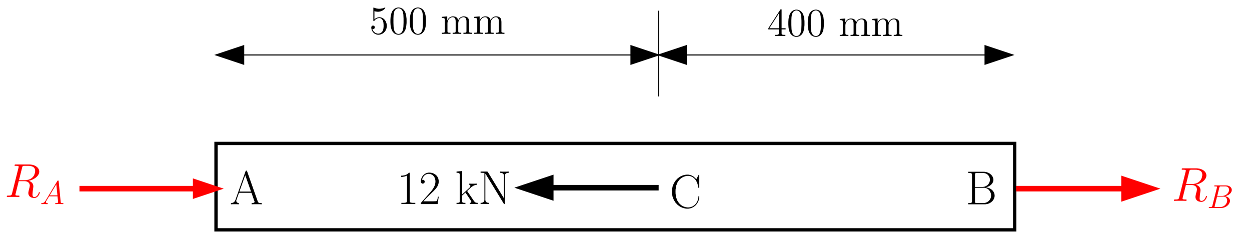
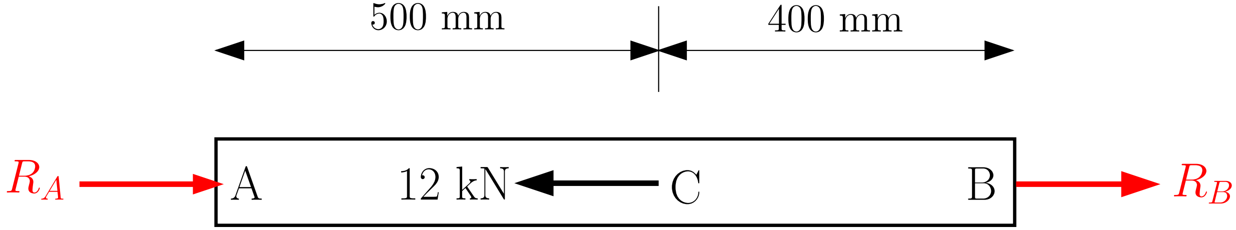
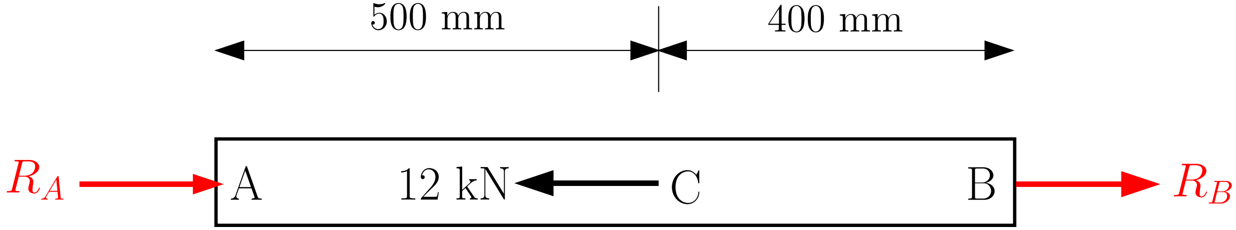


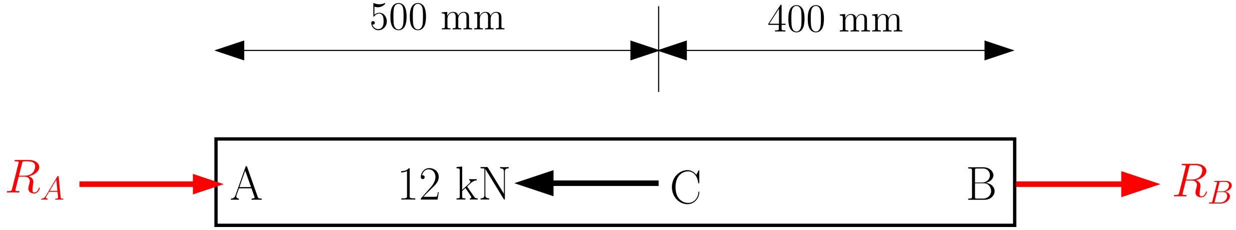

500 mm
400 mm
R_A
A
12 kN
C
B
R_B



$$\Sigma F_x \;=\; R_A + R_B - 12 \;=\; 0$$



$$\begin{aligned} \Sigma F_x &= R_A + R_B - 12 &= 0 \\ \Rightarrow & \qquad R_A + R_B &= 12 \end{aligned}$$



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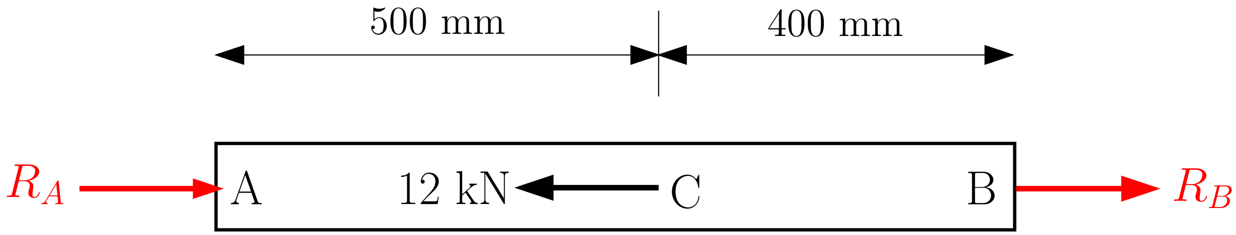
Two unknowns and a single equation; the problem is *statically indeterminate*.



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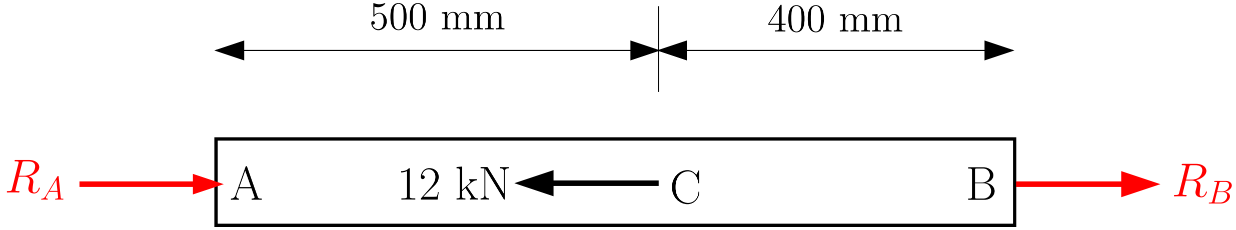
The supports at A and B are fixed so $\delta_{AB} = 0$.



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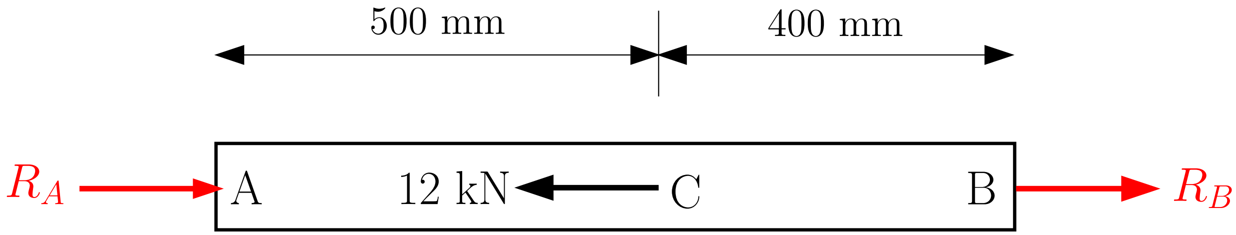

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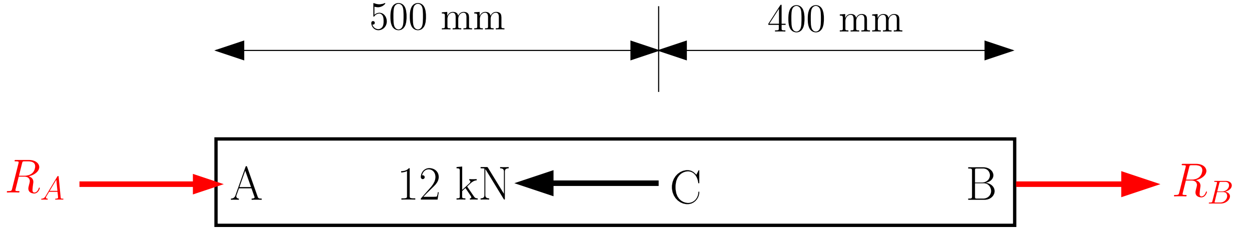
$$\begin{aligned} \Rightarrow \quad \delta_{AC} + \delta_{CB} &= 0 \\ \Rightarrow \quad \frac{-R_A \times 500}{AE} + \frac{R_B \times 400}{AE} &= 0 \end{aligned}$$



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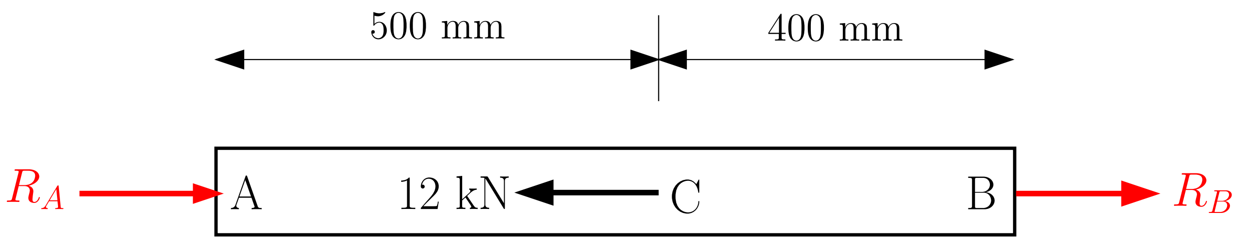

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Now we have two equations and two unknowns; we can solve for R_A and R_B

$$\begin{aligned} R_A + R_B &= 12 \\ 400R_B &= 500R_A \end{aligned}$$

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\Rightarrow \quad 900 R_B &= 6000 \\
\Rightarrow \quad R_B &= 6.667\ \text{kN}
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\Rightarrow \quad R_A &= 5.333\ \text{kN}
\end{aligned}
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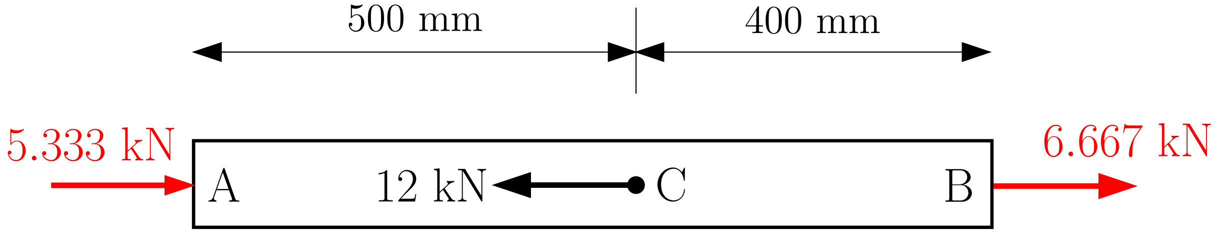
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400 mm
5.333 kN
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A
12 kN
C
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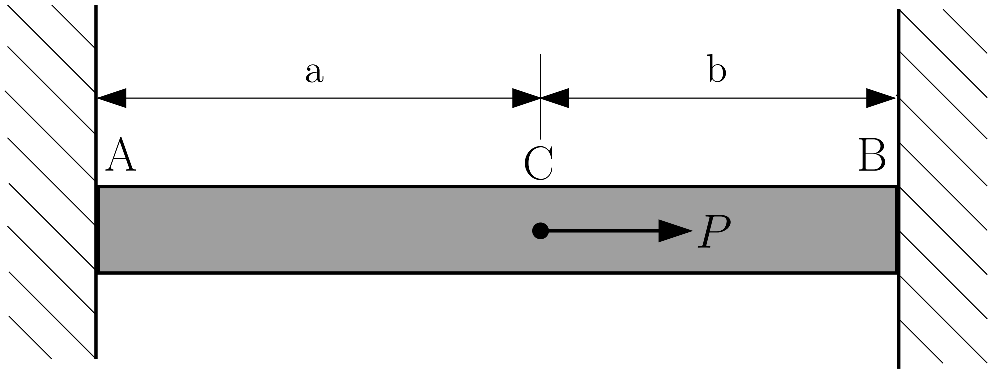
Exercise: Find R_A and R_B for the problem illustrated:



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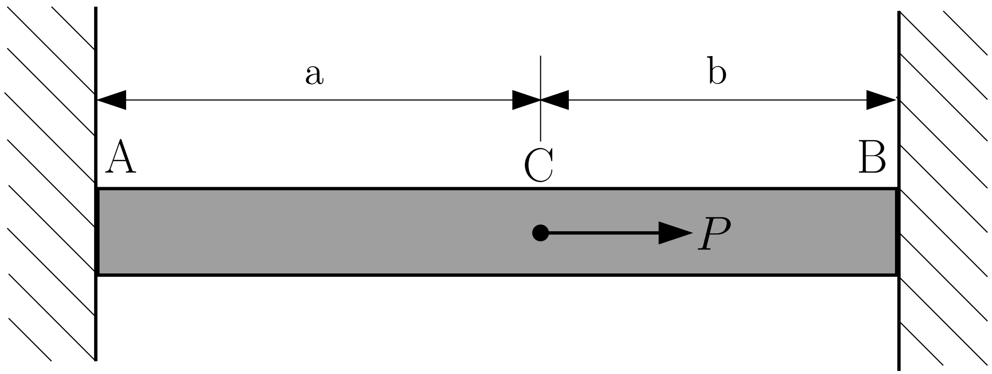

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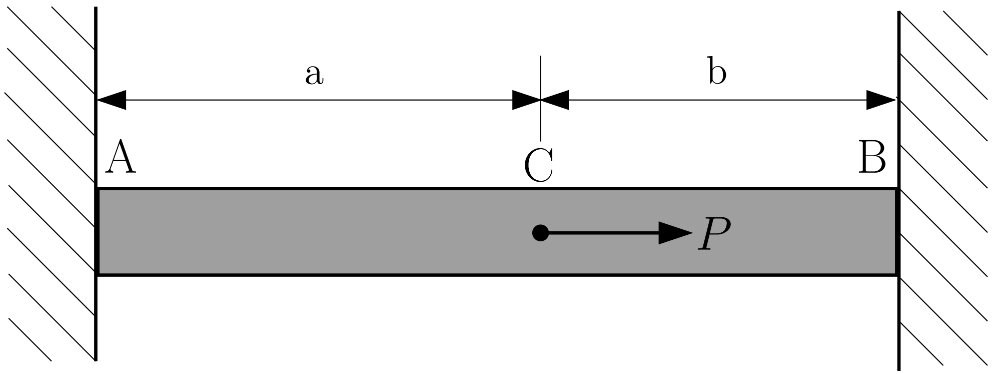

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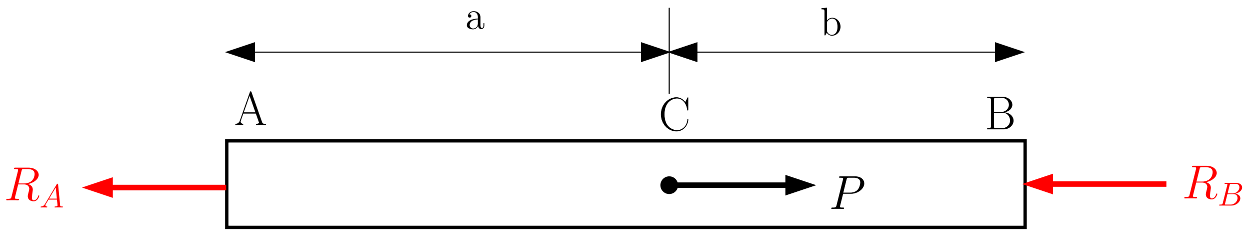
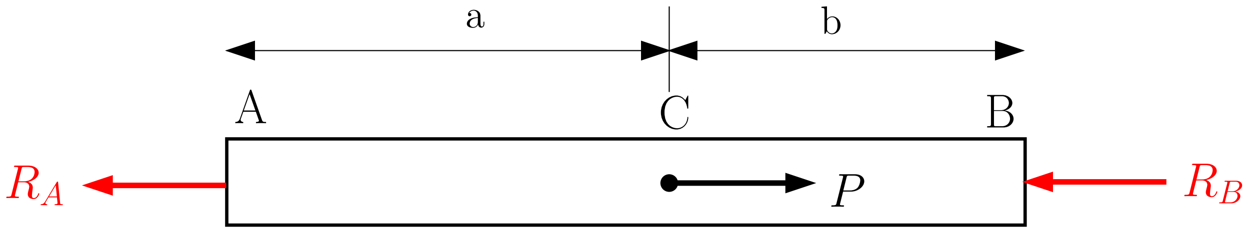
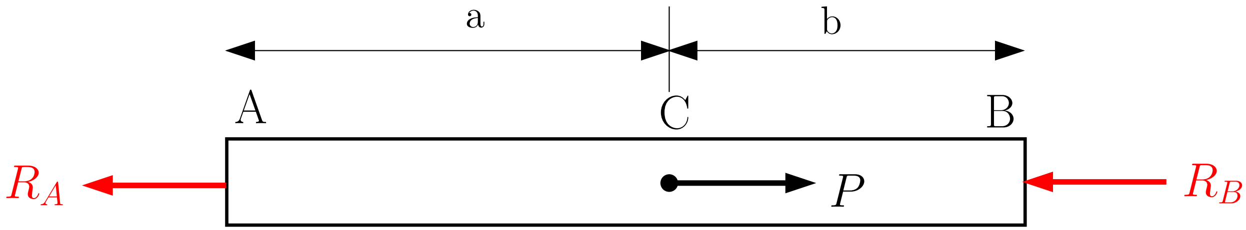


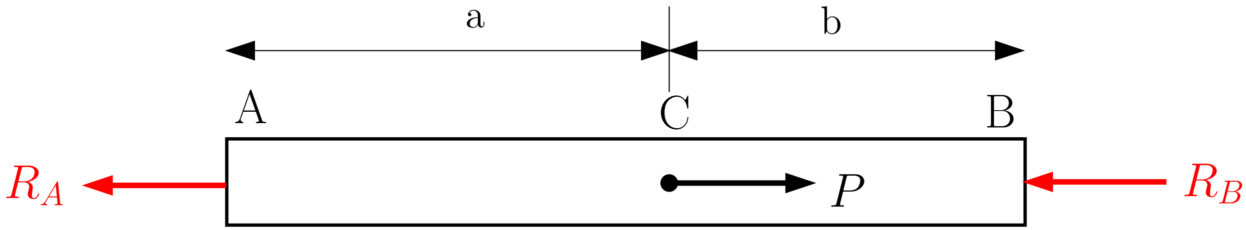


Solution:



Solution:

$$R_A + R_B - P = 0$$



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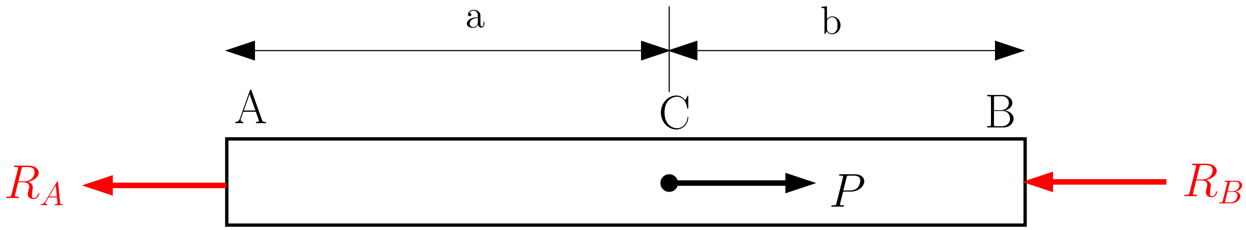

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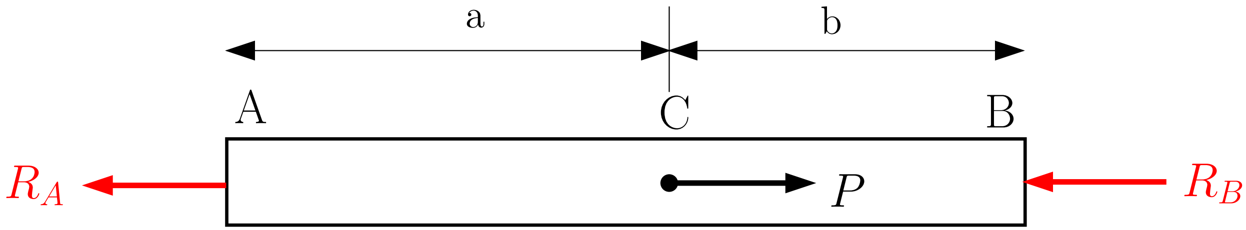

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\begin{aligned}
R_A + R_B - P &= 0 \\
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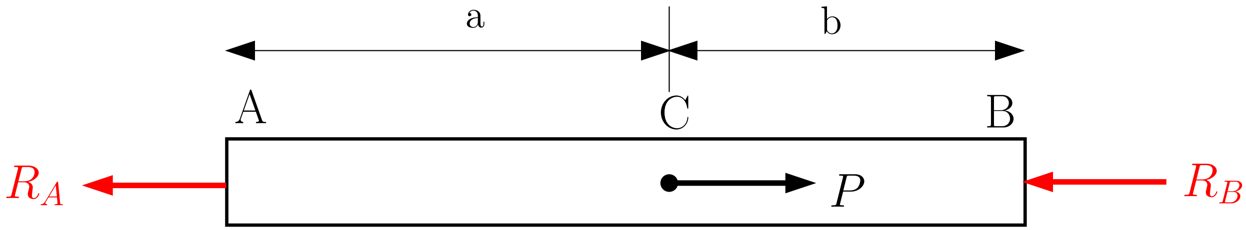
\delta_{AC} + \delta_{CB} &= 0 \\
\Rightarrow \qquad \frac{R_A \times a}{AE} + \frac{-R_B \times b}{AE} &= 0
\end{aligned}
$$



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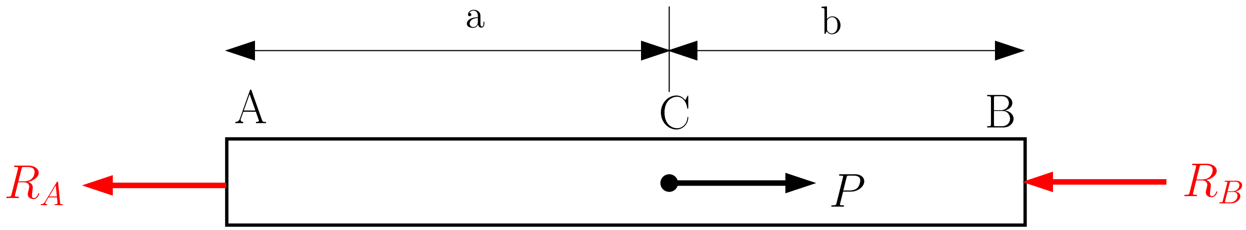
$$\begin{aligned} R_A + R_B - P &= 0 \\ \Rightarrow \quad R_A &= P - R_B \end{aligned}$$

$$\begin{aligned} \delta_{AC} + \delta_{CB} &= 0 \\ \Rightarrow \quad \frac{R_A \times a}{AE} + \frac{-R_B \times b}{AE} &= 0 \\ \Rightarrow \quad aR_A &= bR_B \end{aligned}$$



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\Rightarrow \quad a\left(P - R_B\right) &= bR_B
\end{aligned}
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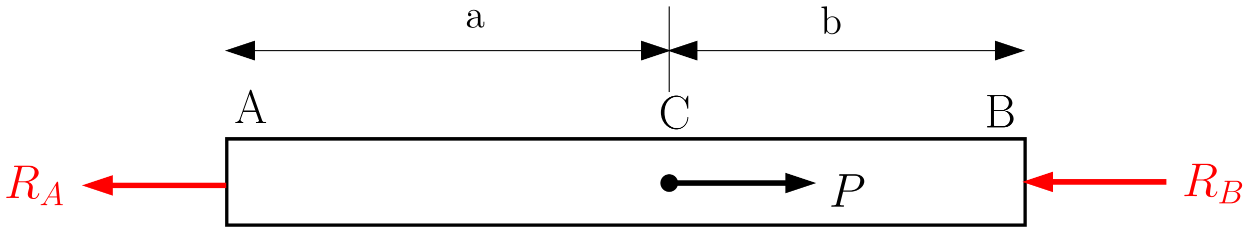

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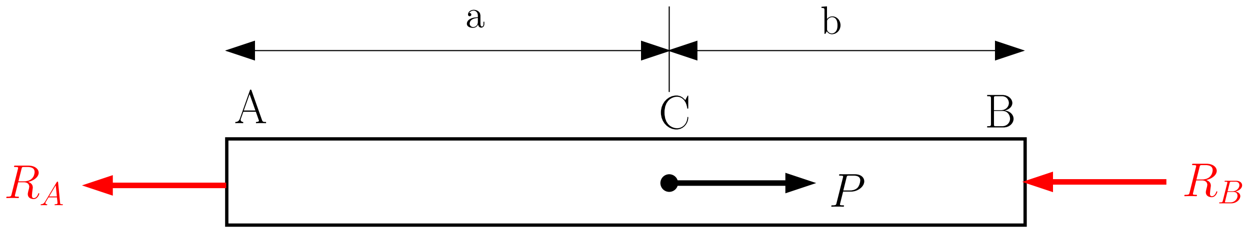

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\Rightarrow \quad aP &= \left(a+b\right) R_B \\
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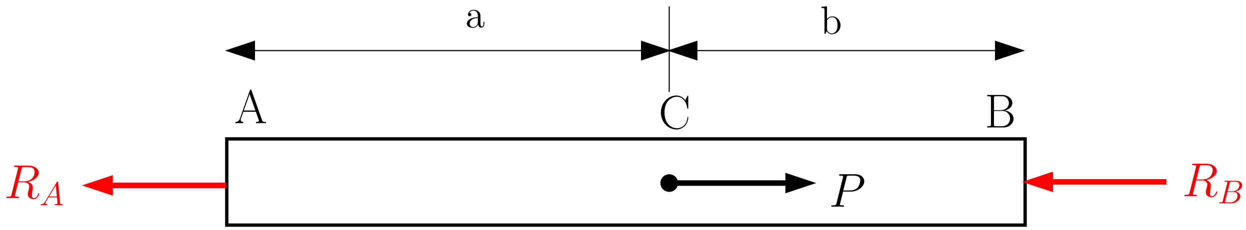
\end{aligned}
$$



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\Rightarrow \quad aP &= (a+b)\,R_B \\
\Rightarrow \quad R_B &= \left(\frac{a}{a+b}\right) P \\
\Rightarrow \quad R_A &= \left(\frac{b}{a+b}\right) P
\end{aligned}
$$

Steel-reinforced concrete is used in the construction of many structures:

- Bridges
- Basements
- High-Rise Buildings
- Stadia, such as the SaddleDome or the Speed-Skating Oval

- Concrete has a high load-bearing capacity in compression but is not very strong under a tensile load.

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- Concrete has a high load-bearing capacity in compression but is not very strong under a tensile load.
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- Combining steel rod and concrete gives a building material with both good tensile and compressive load-bearing qualities.

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    - Under compression, δ is negative and there is negative axial strain

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  - The concrete is under tension laterally
  - Horizontal steel-reinforcing increases the lateral tensile strength of the column

A concrete footing is poured:

- It contains steel rebar throughout
- Steel extrudes from the top of the footing
- This will be attached to the steel for the column.

Steel is tied for the column

A frame is built around the steel and the concrete column is poured

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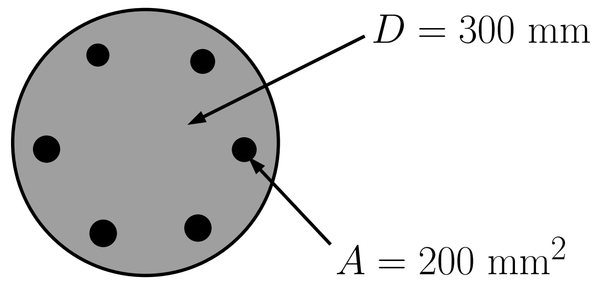
- How can we calculate the deformation of a steel-reinforced concrete column?
  - E_C is not the same as E_S so we cannot simply apply $\delta = \frac{PL}{AE}$ for the whole column
  - We cannot solve this problem directly using the equations of statics, so this is a statically-indeterminate problem

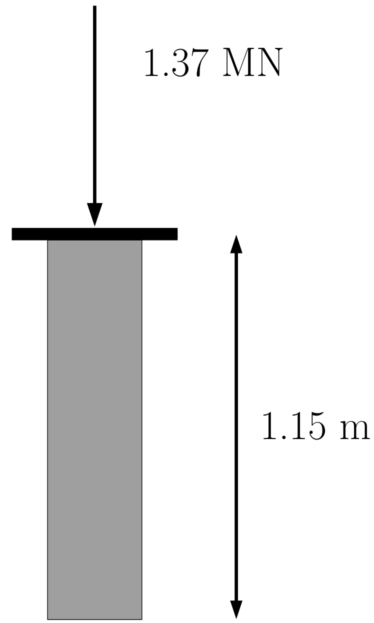
Example: A concrete column has a diameter of 300 mm. The column has 6 steel reinforcing rods.

Each rod has a cross-sectional area of 200 mm^2. (See plan view)

$E_S = 210$ GPa and $E_C = 25$ GPa

The column is 1.15 m long and has a load of 1.37 MN is applied to a rigid steel plate at the top of the column (the plate distributes the load evenly over the top of the column).





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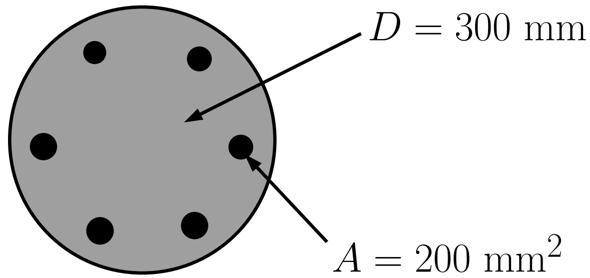
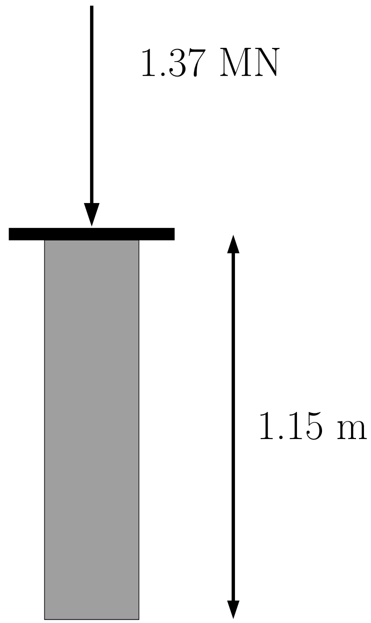
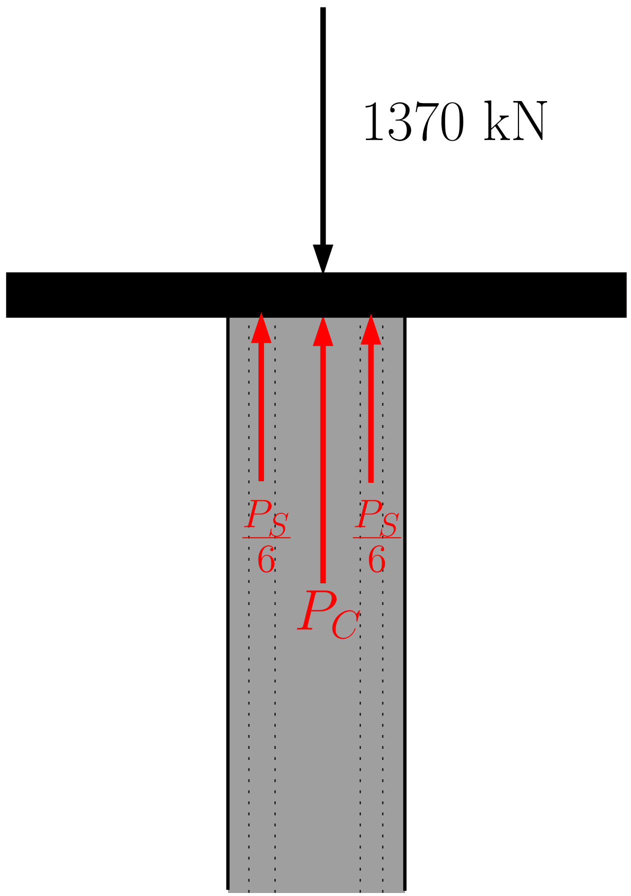
Find the stress in the steel and in the concrete, and the deformation under the load



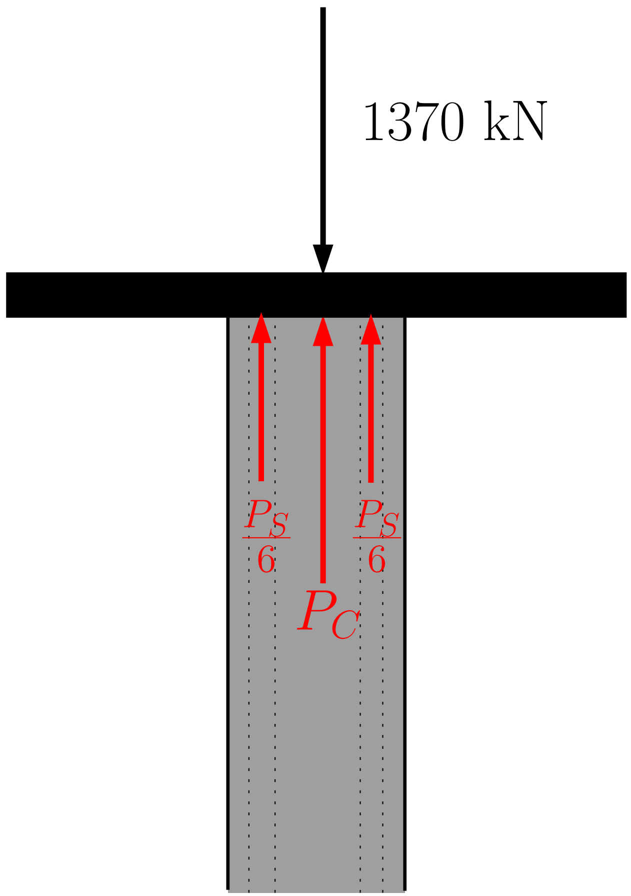


Solution: Let P_S be the total reaction force of the six steel rods and P_C the reaction force of the concrete.



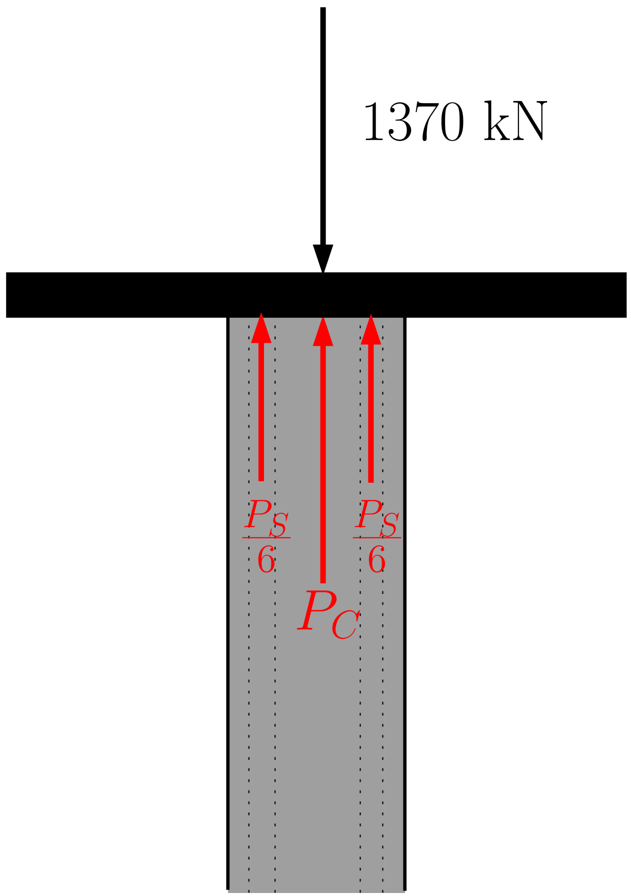
Solution: Let P_S be the total reaction force of the six steel rods and P_C the reaction force of the concrete.

$$\Sigma F_y = P_S + P_C - 1370 = 0$$



Solution: Let P_S be the total reaction force of the six steel rods and P_C the reaction force of the concrete.

$$\begin{aligned} \Sigma F_y &= P_S + P_C - 1370 &= 0 \\ & P_S + P_C &= 1370 \text{ kN} \end{aligned}$$



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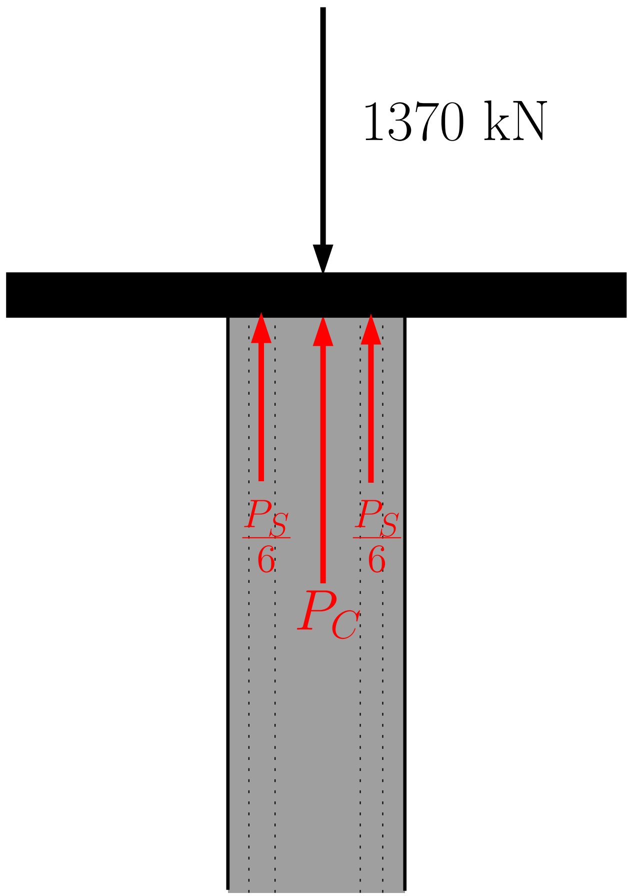
We have a single equation with two unknowns, so the problem is statically indeterminate.



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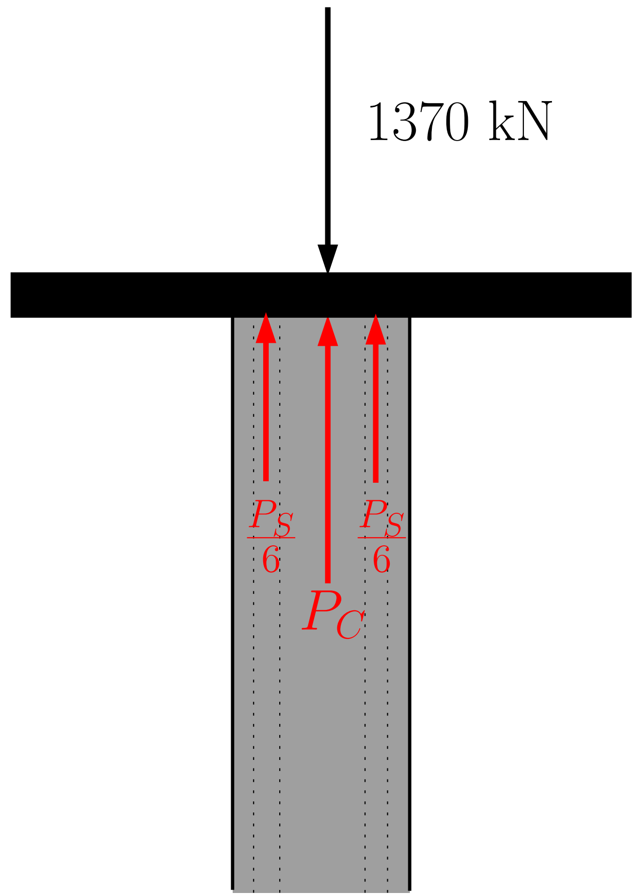

We have a single equation with two unknowns, so the problem is statically indeterminate.
The concrete and the steel rods deform (contract) by the same amount, δ, so...

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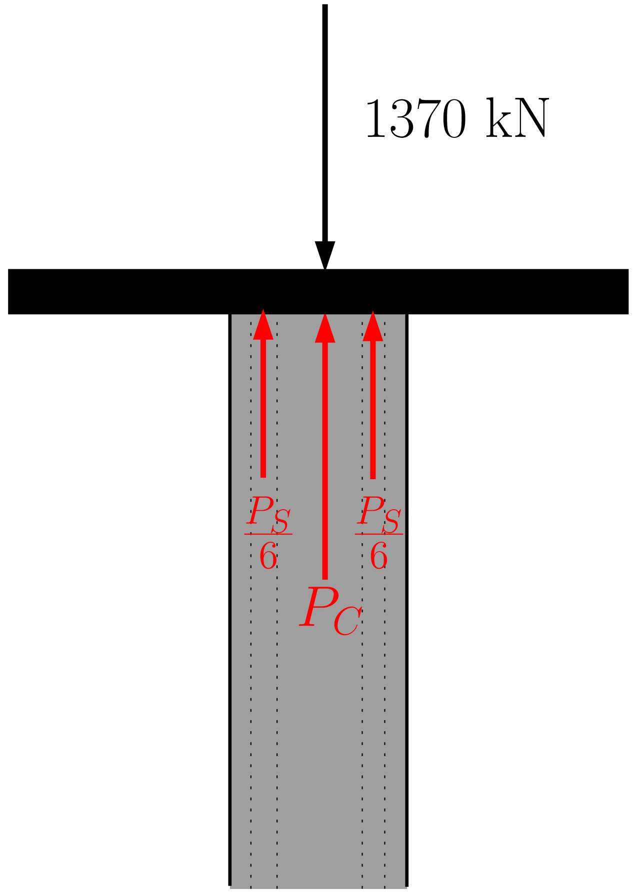
$$\begin{aligned} \Sigma F_y &= P_S + P_C - 1370 &= 0 \\ & P_S + P_C &= 1370 \text{ kN} \end{aligned}$$

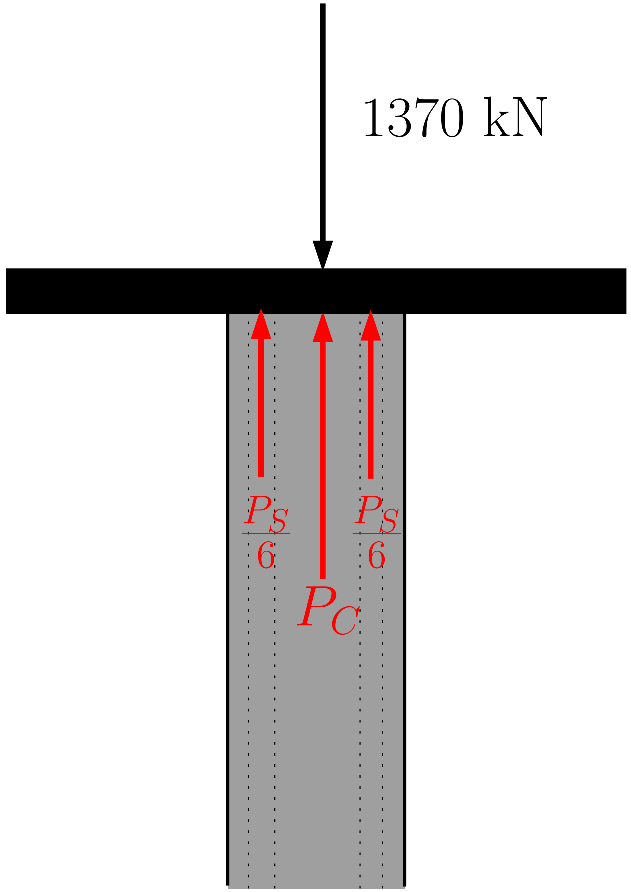


We have a single equation with two unknowns, so the problem is statically indeterminate.
The concrete and the steel rods deform (contract) by the same amount, δ, so...

$$\frac{P_S \cdot L_S}{A_S \cdot E_S} = \delta = \frac{P_C \cdot L_C}{A_C \cdot E_C}$$

Solution:

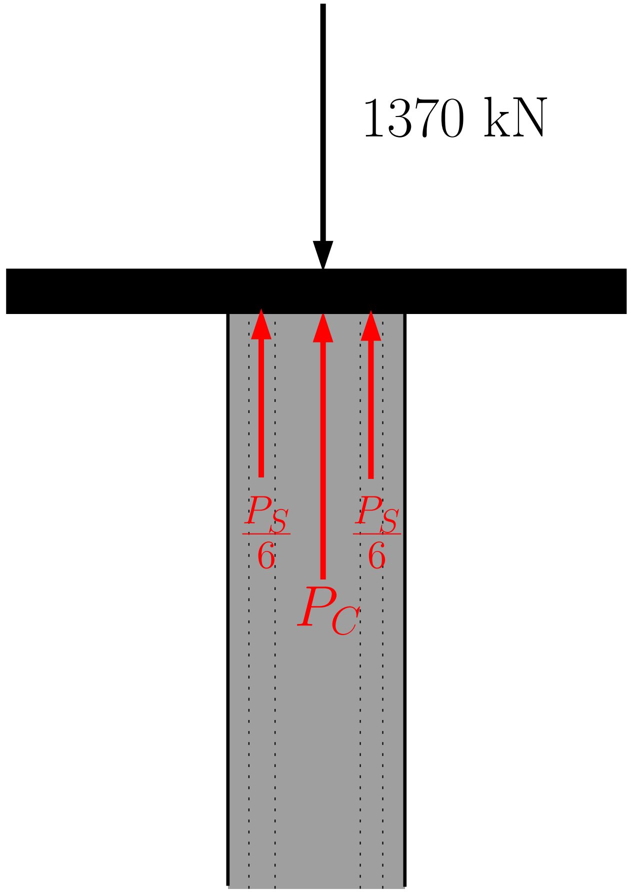
$$\frac{P_S \cdot L_S}{A_S \cdot E_S} = \frac{P_C \cdot L_C}{A_C \cdot E_C}$$



Solution:

$$\frac{P_S \cdot L_S}{A_S \cdot E_S} = \frac{P_C \cdot L_C}{A_C \cdot E_C}$$

$$\Rightarrow \quad \frac{P_S \times 1150}{(6 \times 200) \times E_S} = \frac{P_C \times 1150}{\left(\frac{\pi \times 300^2}{4} - (6 \times 200)\right) \times E_C}$$



Solution:

$$\frac{P_S \cdot L_S}{A_S \cdot E_S} = \frac{P_C \cdot L_C}{A_C \cdot E_C}$$

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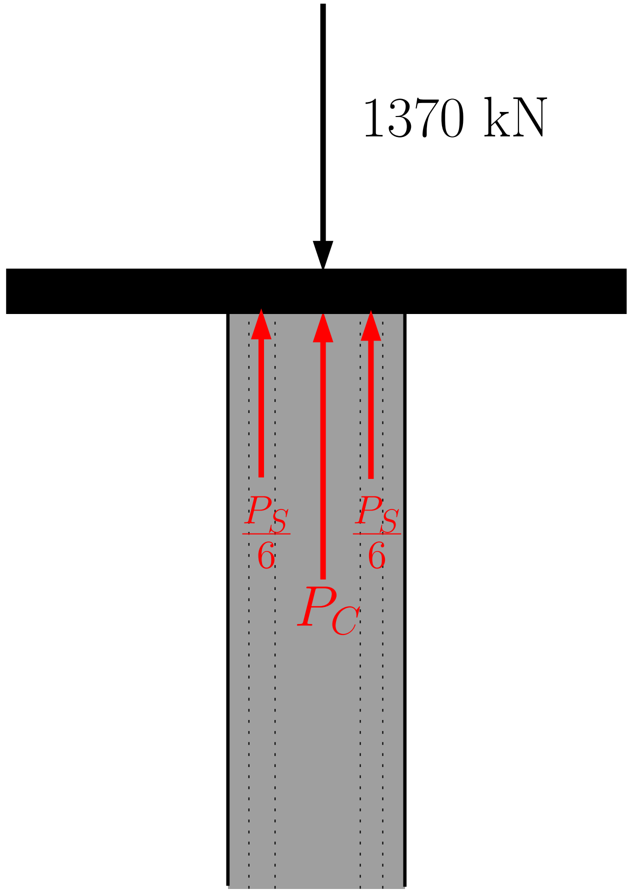
$$\Rightarrow \quad \frac{P_S \times 1150}{1200 \times (200 \times 10^3)} = \frac{P_C \times 1150}{\left(\frac{\pi \times 300^2}{4} - 1200\right) \times (25 \times 10^3)}$$



Solution:

$$\frac{P_S \cdot L_S}{A_S \cdot E_S} = \frac{P_C \cdot L_C}{A_C \cdot E_C}$$

$$\Rightarrow \quad \frac{P_S \times 1150}{(6\times 200)\times E_S} = \frac{P_C \times 1150}{\left(\frac{\pi \times 300^2}{4} - (6\times 200)\right)\times E_C}$$

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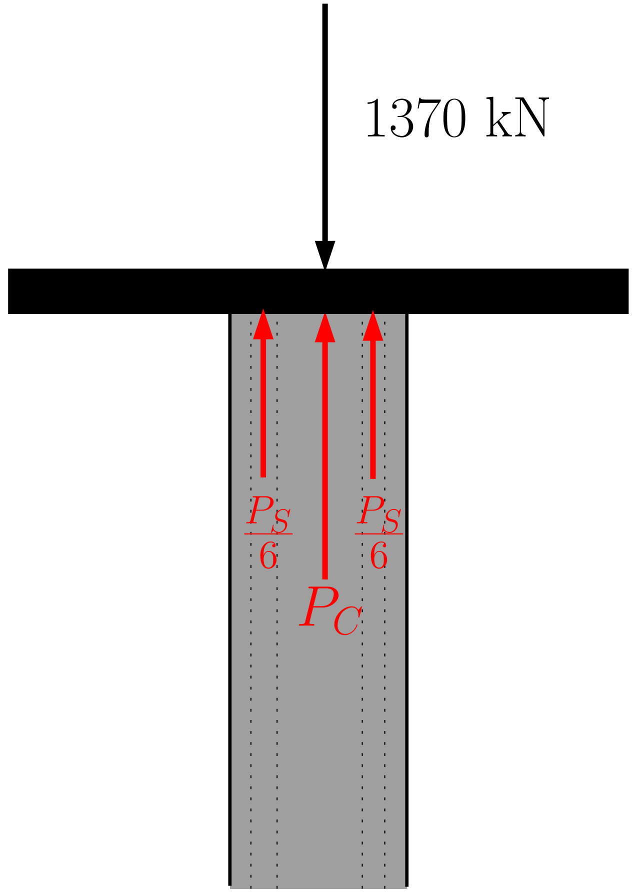
$$\Rightarrow \quad \frac{P_S}{1200\times 200} = \frac{P_C}{69486\times 25}$$



Solution:

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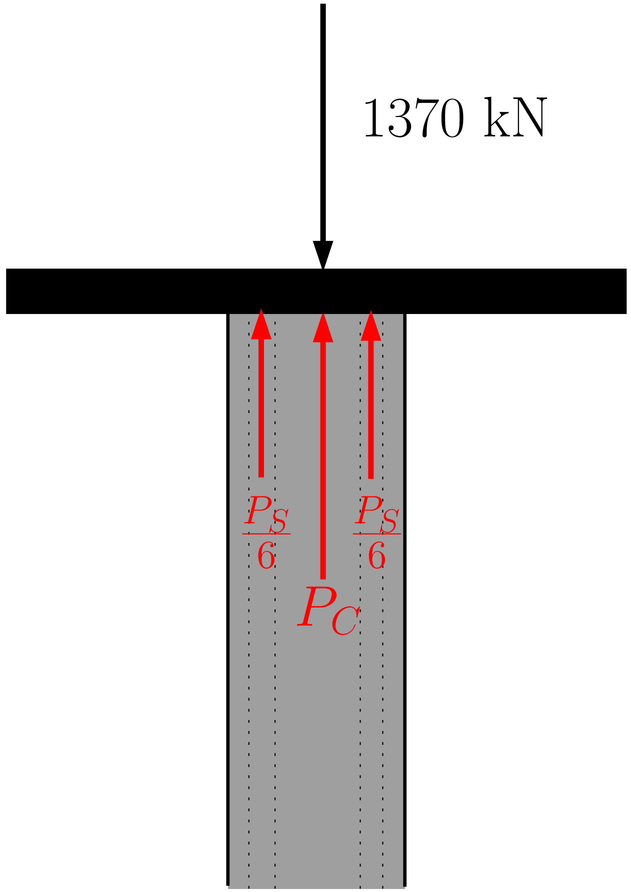
$$\Rightarrow \quad \frac{P_S}{1200\times 200} = \frac{P_C}{69486\times 25}$$

$$\Rightarrow \quad P_S = \frac{1200\times 200}{69486\times 25}\cdot P_C$$



Solution:

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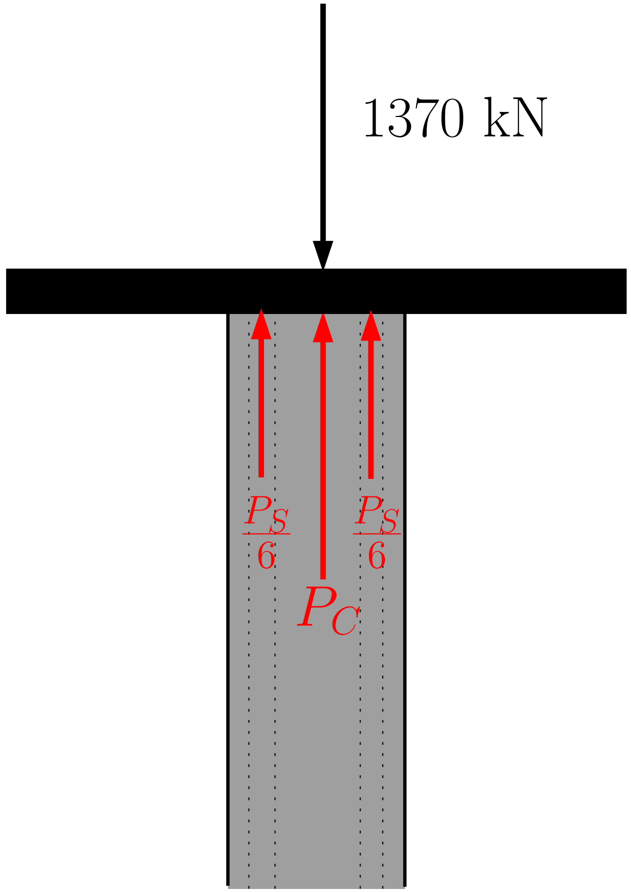
$$\Rightarrow \quad P_S = \frac{1200\times 200}{69486\times 25}\cdot P_C$$

$$\Rightarrow \quad P_S = 0.13816 P_C$$



Solution:

$$\frac{P_S \cdot L_S}{A_S \cdot E_S} = \frac{P_C \cdot L_C}{A_C \cdot E_C}$$

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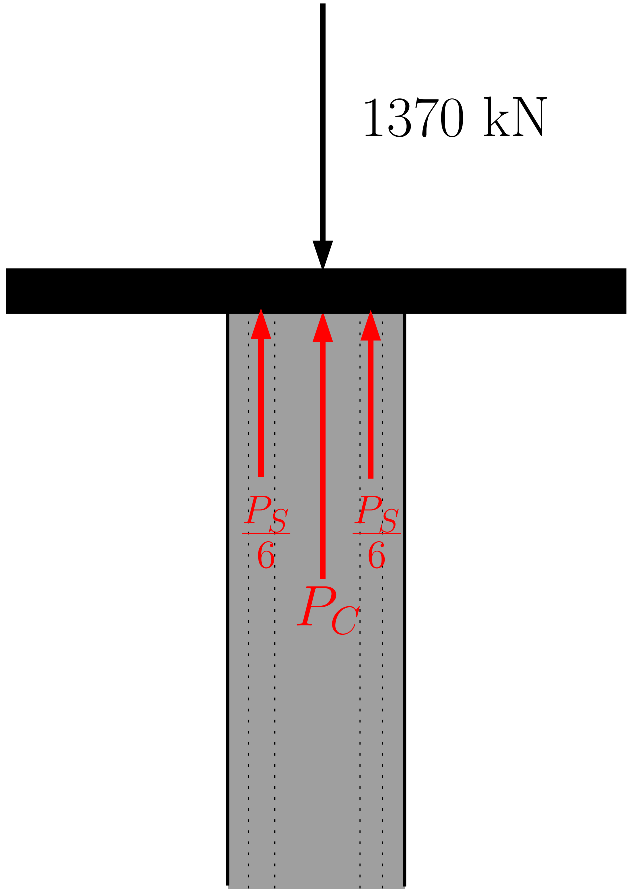
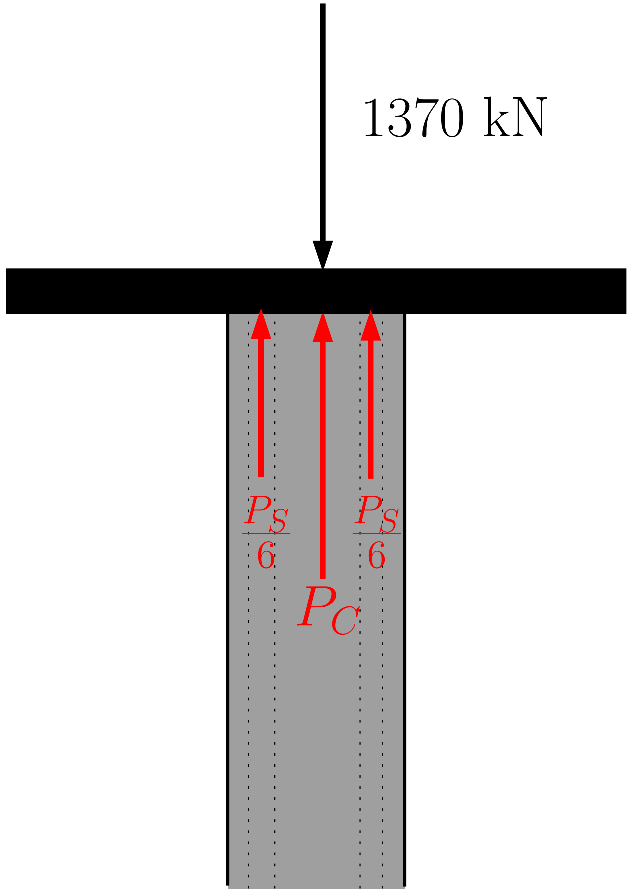


We now have two equations for the two unknowns, P_S and P_C.

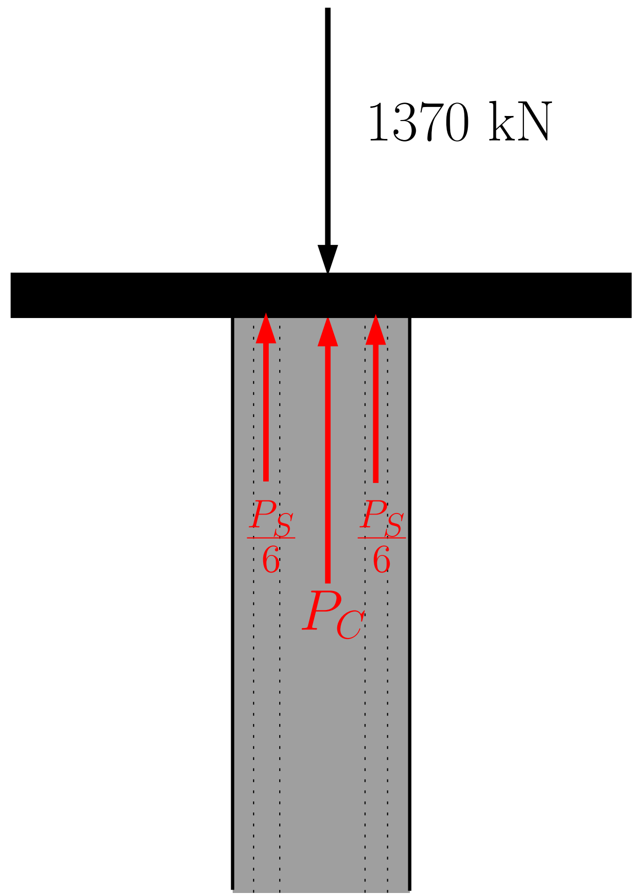
Solution:

$$\begin{aligned} P_S + P_C &= 1370 \\ P_S &= 0.13816 P_C \end{aligned}$$



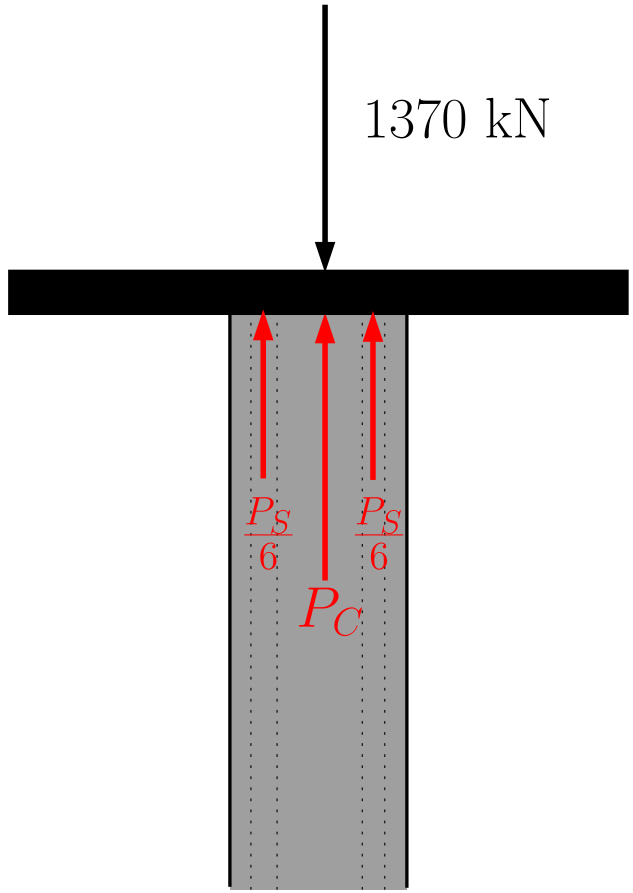
Solution:

$$
\begin{aligned}
P_S + P_C &= 1370 \\
P_S &= 0.13816 P_C \\
\Rightarrow \quad 0.13816 P_C + P_C &= 1370
\end{aligned}
$$



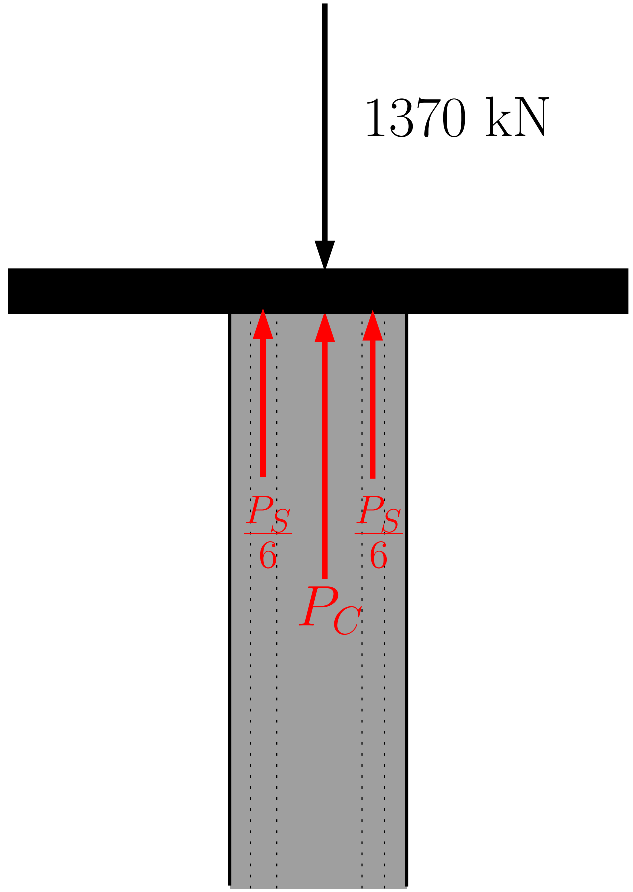
Solution:

$$
\begin{aligned}
P_S + P_C &= 1370 \\
P_S &= 0.13816 P_C \\
\Rightarrow \quad 0.13816 P_C + P_C &= 1370 \\
\Rightarrow \quad P_C &= \tfrac{1370}{1+0.13816}
\end{aligned}
$$



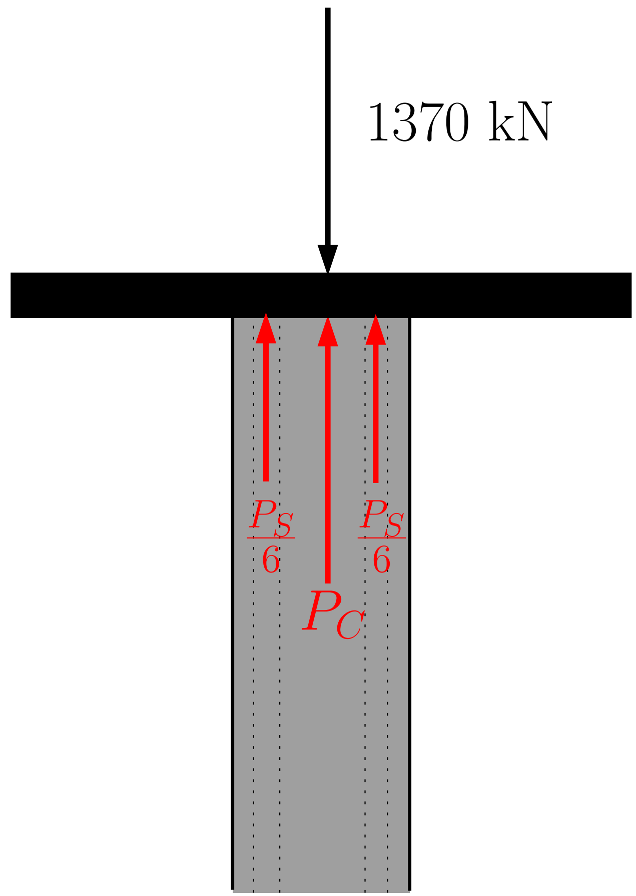
Solution:

$$
\begin{aligned}
P_S + P_C &= 1370 \\
P_S &= 0.13816 P_C \\
\Rightarrow \quad 0.13816 P_C + P_C &= 1370 \\
\Rightarrow \quad P_C &= \frac{1370}{1+0.13816} \\
\Rightarrow \quad P_C &= 1203.7 \text{ kN}
\end{aligned}
$$



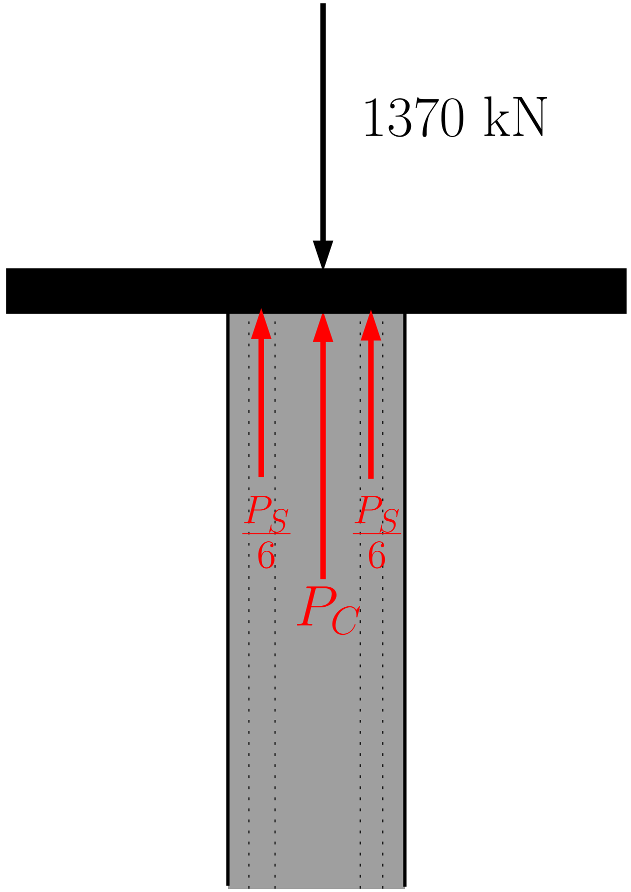
Solution:

$$
\begin{aligned}
P_S + P_C &= 1370 \\
P_S &= 0.13816 P_C \\
\Rightarrow \quad 0.13816 P_C + P_C &= 1370 \\
\Rightarrow \quad P_C &= \frac{1370}{1+0.13816} \\
\Rightarrow \quad P_C &= 1203.7\ \text{kN} \\
\Rightarrow \quad P_S &= 166.3\ \text{kN}
\end{aligned}
$$



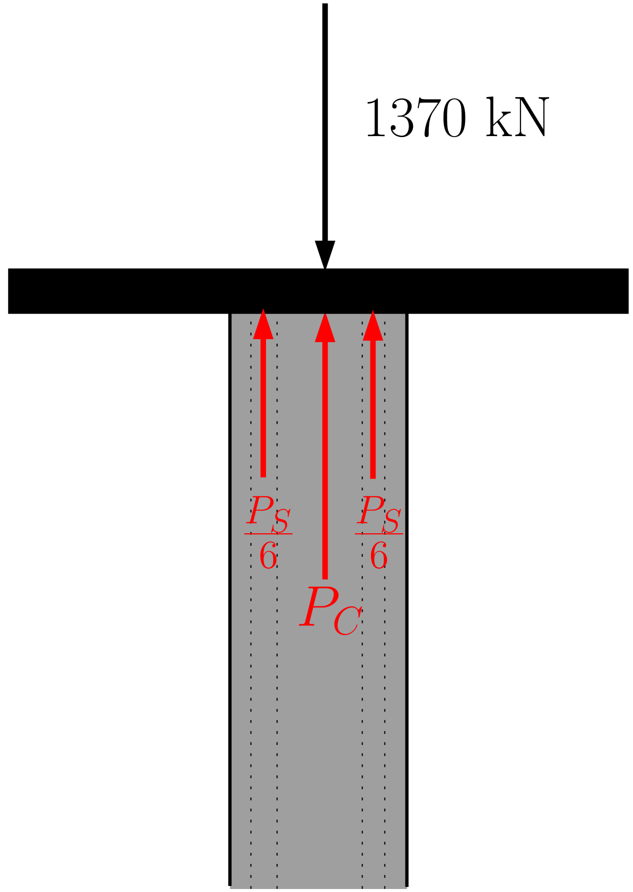
Solution:

$$\begin{aligned} P_S + P_C &= 1370 \\ P_S &= 0.13816 P_C \\ \Rightarrow \quad 0.13816 P_C + P_C &= 1370 \\ \Rightarrow \quad P_C &= \tfrac{1370}{1+0.13816} \\ \Rightarrow \quad P_C &= 1203.7 \text{ kN} \\ \Rightarrow \quad P_S &= 166.3 \text{ kN} \end{aligned}$$



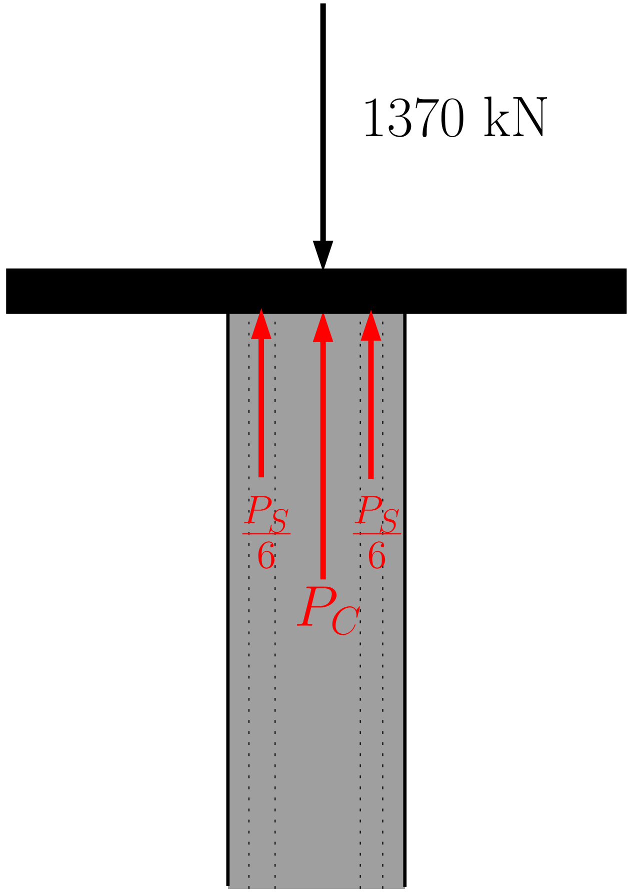
We can now calculate the stress in the steel and in the concrete

Solution: Find the stress in the concrete:



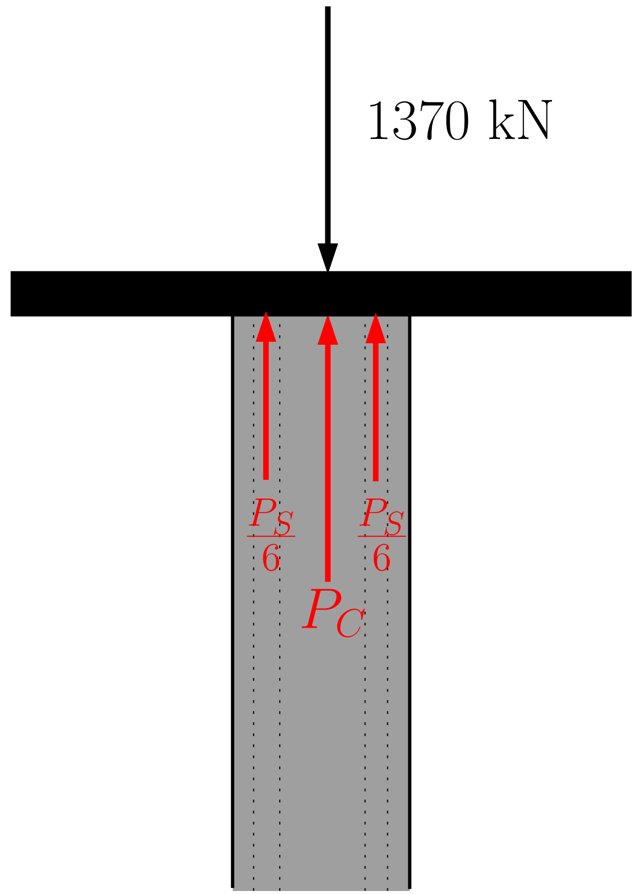
Solution: Find the stress in the concrete:

$$P_C \quad = \quad 1098 \text{ kN}$$



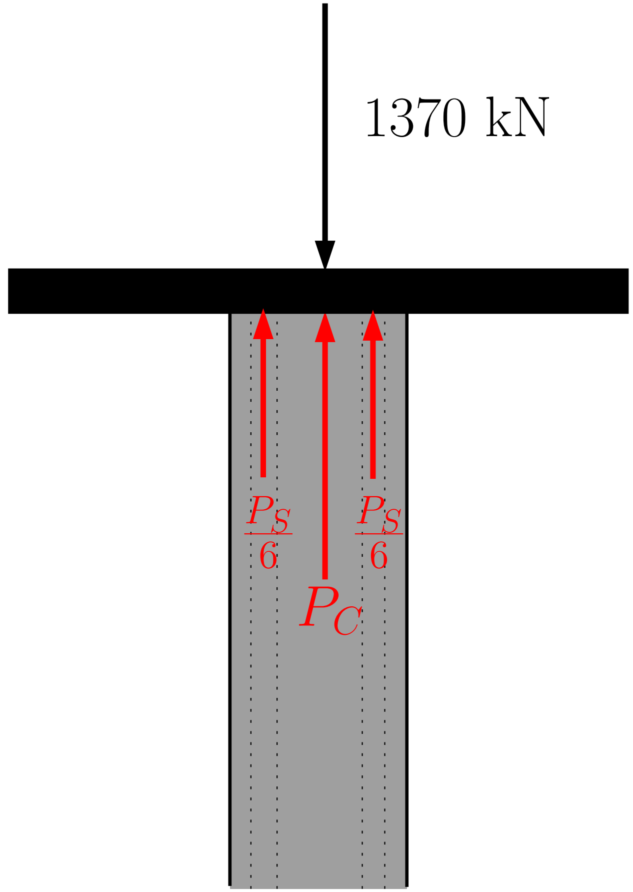
Solution: Find the stress in the concrete:

$$
\begin{aligned}
P_C &= 1098\ \text{kN} \\
\Rightarrow \quad \sigma_C &= \frac{P_C}{A}
\end{aligned}
$$



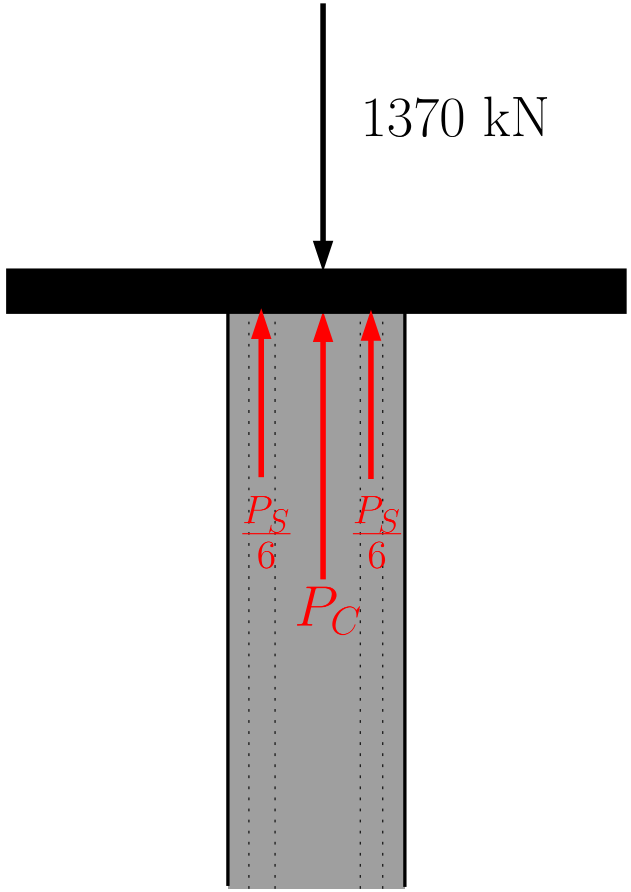
Solution: Find the stress in the concrete:

$$
\begin{aligned}
P_C &= 1098 \text{ kN} \\
\Rightarrow \quad \sigma_C &= \frac{P_C}{A} \\
\Rightarrow \quad \sigma_C &= \frac{1098}{\frac{\pi \times 300^2}{4} - (6 \times 200)}
\end{aligned}
$$



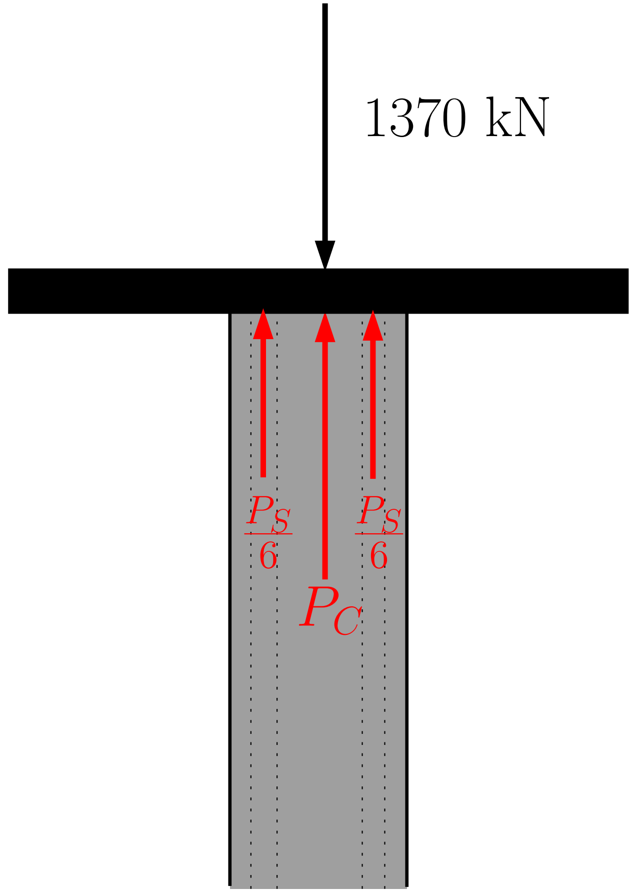
Solution: Find the stress in the concrete:

$$
\begin{aligned}
& P_C && = && 1098\ \mathrm{kN} \\
\Rightarrow\ & \sigma_C && = && \frac{P_C}{A} \\
\Rightarrow\ & \sigma_C && = && \frac{1098}{\frac{\pi \times 300^2}{4} - (6 \times 200)} \\
\Rightarrow\ & \sigma_C && = && 0.0580\ \frac{\mathrm{kN}}{\mathrm{mm}^2}
\end{aligned}
$$



Solution: Find the stress in the concrete:

$$
\begin{aligned}
& P_C &=& \ 1098 \text{ kN} \\
\Rightarrow\ & \sigma_C &=& \ \frac{P_C}{A} \\
\Rightarrow\ & \sigma_C &=& \ \frac{1098}{\frac{\pi \times 300^2}{4} - (6 \times 200)} \\
\Rightarrow\ & \sigma_C &=& \ 0.0580 \ \frac{\text{kN}}{\text{mm}^2} \\
\Rightarrow\ & \sigma_C &=& \ 58.0 \text{ MPa}
\end{aligned}
$$



Solution: Find the stress in the concrete:

$$
\begin{aligned}
 & P_C &=& \ 1098 \text{ kN} \\
\Rightarrow & \sigma_C &=& \ \frac{P_C}{A} \\
\Rightarrow & \sigma_C &=& \ \frac{1098}{\frac{\pi\times300^2}{4}-(6\times200)} \\
\Rightarrow & \sigma_C &=& \ 0.0580 \ \frac{\text{kN}}{\text{mm}^2} \\
\Rightarrow & \sigma_C &=& \ 58.0 \text{ MPa}
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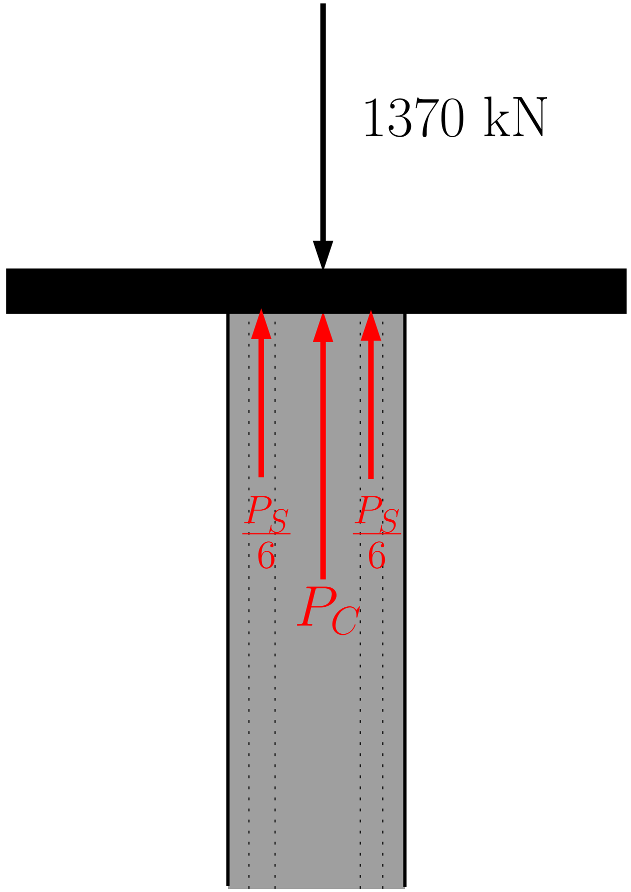
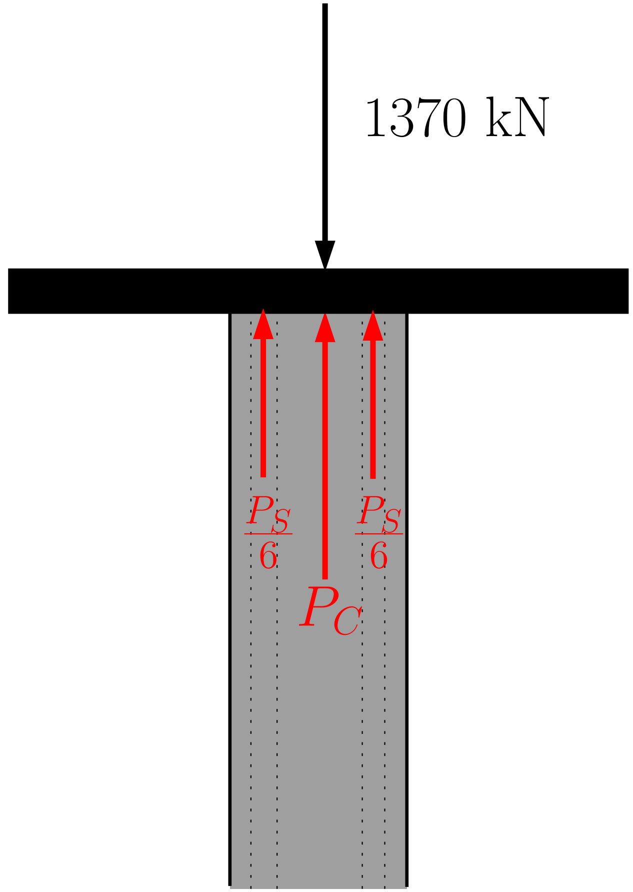
$$



Find the stress in the steel:

Solution: Find the stress in the concrete:

$$
\begin{aligned}
P_C &= 1098 \text{ kN} \\
\Rightarrow \quad \sigma_C &= \frac{P_C}{A} \\
\Rightarrow \quad \sigma_C &= \frac{1098}{\frac{\pi \times 300^2}{4} - (6 \times 200)} \\
\Rightarrow \quad \sigma_C &= 0.0580 \ \frac{\text{kN}}{\text{mm}^2} \\
\Rightarrow \quad \sigma_C &= 58.0 \text{ MPa}
\end{aligned}
$$



Find the stress in the steel:

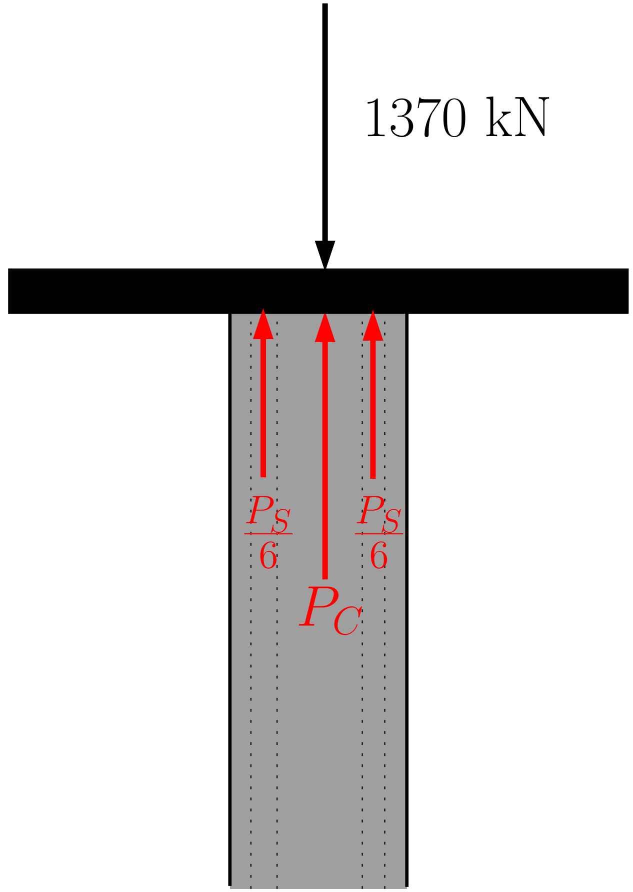
$$P_S = 152 \text{ kN}$$

Solution: Find the stress in the concrete:

$$
\begin{aligned}
P_C &= 1098 \text{ kN} \\
\Rightarrow \quad \sigma_C &= \frac{P_C}{A} \\
\Rightarrow \quad \sigma_C &= \frac{1098}{\frac{\pi \times 300^2}{4} - (6 \times 200)} \\
\Rightarrow \quad \sigma_C &= 0.0580 \ \frac{\text{kN}}{\text{mm}^2} \\
\Rightarrow \quad \sigma_C &= 58.0 \text{ MPa}
\end{aligned}
$$

Find the stress in the steel:

$$
\begin{aligned}
P_S &= 152 \text{ kN} \\
\Rightarrow \quad \sigma_S &= \frac{152}{(6 \times 200)}
\end{aligned}
$$

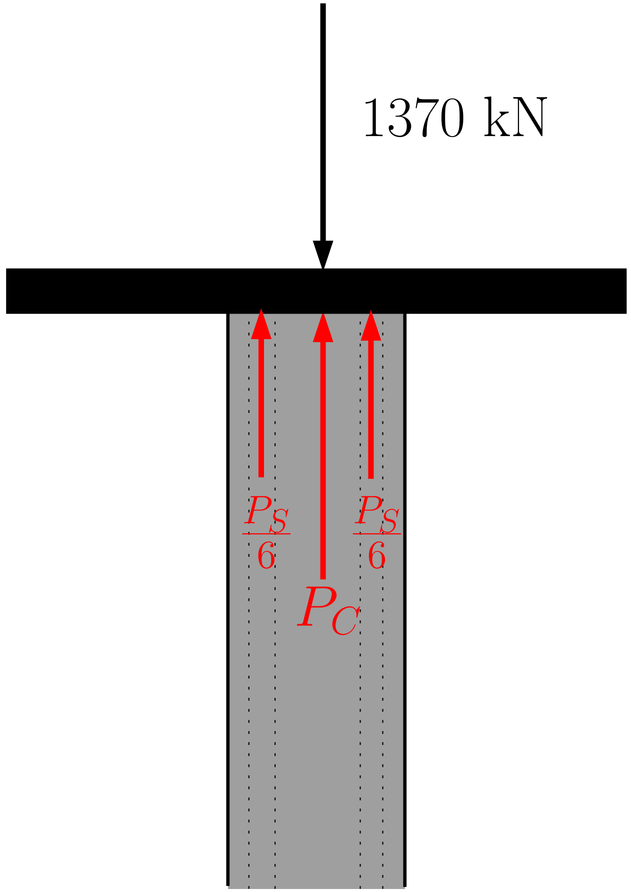


Solution: Find the stress in the concrete:

$$
\begin{aligned}
& P_C &=& \quad 1098 \text{ kN} \\
\Rightarrow\quad & \sigma_C &=& \quad \frac{P_C}{A} \\
\Rightarrow\quad & \sigma_C &=& \quad \frac{1098}{\frac{\pi\times 300^2}{4}-(6\times 200)} \\
\Rightarrow\quad & \sigma_C &=& \quad 0.0580\ \frac{\text{kN}}{\text{mm}^2} \\
\Rightarrow\quad & \sigma_C &=& \quad 58.0 \text{ MPa}
\end{aligned}
$$

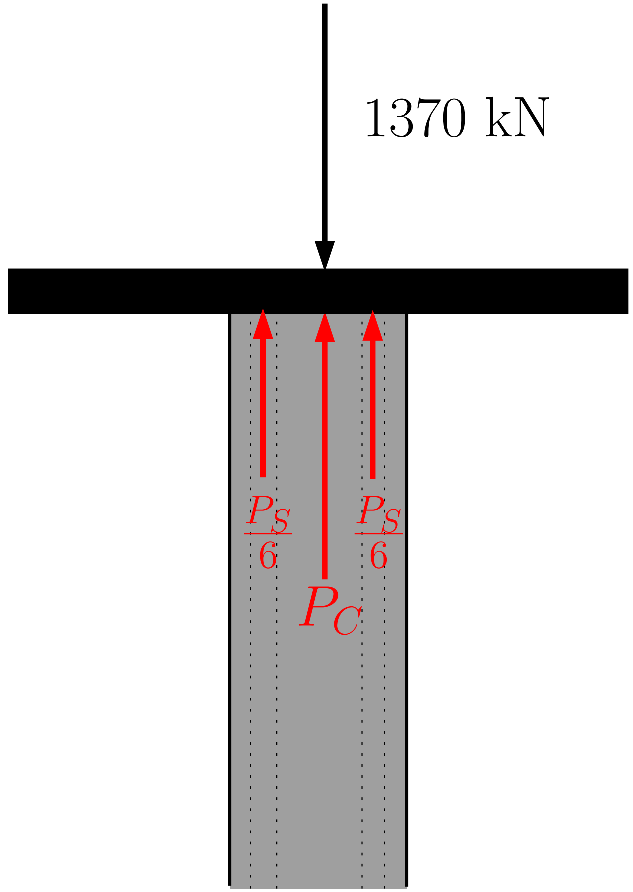
Find the stress in the steel:

$$
\begin{aligned}
& P_S &=& \quad 152 \text{ kN} \\
\Rightarrow\quad & \sigma_S &=& \quad \frac{152}{(6\times 200)} \\
\Rightarrow\quad & \sigma_S &=& \quad 0.1267\ \frac{\text{kN}}{\text{mm}^2}
\end{aligned}
$$



Solution: Find the stress in the concrete:

$$
\begin{aligned}
 & P_C &=& \; 1098 \text{ kN} \\
\Rightarrow & \sigma_C &=& \; \frac{P_C}{A} \\
\Rightarrow & \sigma_C &=& \; \frac{1098}{\frac{\pi \times 300^2}{4} - (6 \times 200)} \\
\Rightarrow & \sigma_C &=& \; 0.0580 \; \frac{\text{kN}}{\text{mm}^2} \\
\Rightarrow & \sigma_C &=& \; 58.0 \text{ MPa}
\end{aligned}
$$



Find the stress in the steel:

$$
\begin{aligned}
 & P_S &=& \; 152 \text{ kN} \\
\Rightarrow & \sigma_S &=& \; \frac{152}{(6 \times 200)} \\
\Rightarrow & \sigma_S &=& \; 0.1267 \; \frac{\text{kN}}{\text{mm}^2} \\
\Rightarrow & \sigma_S &=& \; 126.7 \text{ MPa}
\end{aligned}
$$

Solution: Find the deformation in the concrete:

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$$\delta_C = \frac{P_C \cdot L_C}{A_C \cdot E_C}$$

Solution: Find the deformation in the concrete:

$$\begin{aligned}\delta_C &= \frac{P_C \cdot L_C}{A_C \cdot E_C} \\ \Rightarrow \quad \delta_C &= \frac{1098 \times \left(3.5 \times 10^3\right)}{\left(\frac{\pi \times 300^2}{4} - 1200\right) \times \left(25 \times 10^3\right)}\end{aligned}$$

Solution: Find the deformation in the concrete:

$$\begin{aligned}
\delta_C &= \frac{P_C \cdot L_C}{A_C \cdot E_C} \\
\Rightarrow \quad \delta_C &= \frac{1098 \times \left(3.5 \times 10^3\right)}{\left(\frac{\pi \times 300^2}{4} - 1200\right) \times \left(25 \times 10^3\right)} \\
\Rightarrow \quad \delta_C &= 0.00221 \text{ mm}
\end{aligned}$$

Solution: Find the deformation in the concrete:

$$\begin{aligned} \delta_C &= \frac{P_C \cdot L_C}{A_C \cdot E_C} \\ \Rightarrow \quad \delta_C &= \frac{1098 \times \left(3.5 \times 10^3\right)}{\left(\frac{\pi \times 300^2}{4} - 1200\right) \times \left(25 \times 10^3\right)} \\ \Rightarrow \quad \delta_C &= 0.00221 \text{ mm} \end{aligned}$$

Find the deformation in the steel (if we've done our calculations correctly, then $\delta_S = \delta_C$):

$$\delta_S = \frac{P_S \cdot L_S}{A_S \cdot E_S}$$

Solution: Find the deformation in the concrete:

$$\begin{aligned} \delta_C &= \frac{P_C \cdot L_C}{A_C \cdot E_C} \\ \Rightarrow \quad \delta_C &= \frac{1098 \times \left(3.5 \times 10^3\right)}{\left(\frac{\pi \times 300^2}{4} - 1200\right) \times \left(25 \times 10^3\right)} \\ \Rightarrow \quad \delta_C &= 0.00221 \text{ mm} \end{aligned}$$

Find the deformation in the steel (if we've done our calculations correctly, then $\delta_S = \delta_C$):

$$\begin{aligned} \delta_S &= \frac{P_S \cdot L_S}{A_S \cdot E_S} \\ \Rightarrow \quad \delta_S &= \frac{152 \times \left(3.5 \times 10^3\right)}{1200 \times \left(200 \times 10^3\right)} \end{aligned}$$

Solution: Find the deformation in the concrete:

$$\begin{aligned}
\delta_C &= \frac{P_C \cdot L_C}{A_C \cdot E_C} \\
\Rightarrow \quad \delta_C &= \frac{1098 \times \left(3.5 \times 10^3\right)}{\left(\frac{\pi \times 300^2}{4} - 1200\right) \times \left(25 \times 10^3\right)} \\
\Rightarrow \quad \delta_C &= 0.00221 \text{ mm}
\end{aligned}$$

Find the deformation in the steel (if we've done our calculations correctly, then $\delta_S = \delta_C$):

$$\begin{aligned}
\delta_S &= \frac{P_S \cdot L_S}{A_S \cdot E_S} \\
\Rightarrow \quad \delta_S &= \frac{152 \times \left(3.5 \times 10^3\right)}{1200 \times \left(200 \times 10^3\right)} \\
\Rightarrow \quad \delta_S &= 0.00222 \text{ mm}
\end{aligned}$$

Solution: Find the deformation in the concrete:

$$\begin{aligned}
\delta_C &= \frac{P_C \cdot L_C}{A_C \cdot E_C} \\
\Rightarrow \quad \delta_C &= \frac{1098 \times \left(3.5 \times 10^3\right)}{\left(\frac{\pi \times 300^2}{4} - 1200\right) \times \left(25 \times 10^3\right)} \\
\Rightarrow \quad \delta_C &= 0.00221 \text{ mm}
\end{aligned}$$

Find the deformation in the steel (if we've done our calculations correctly, then $\delta_S = \delta_C$):

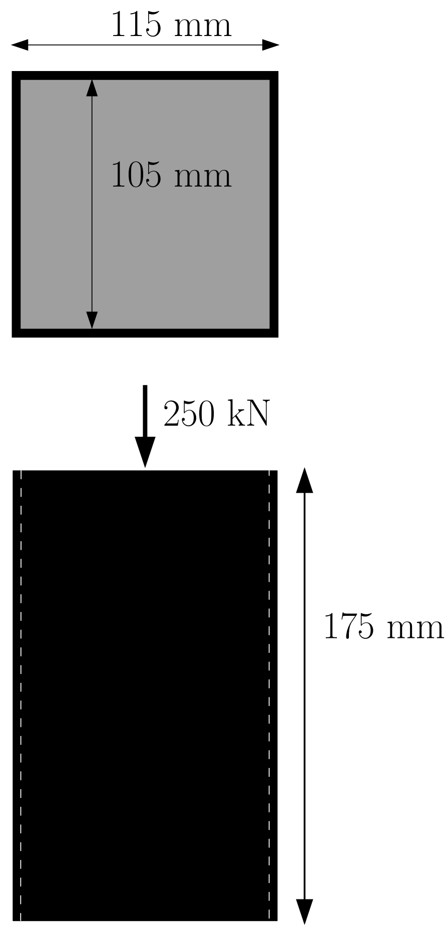
$$\begin{aligned}
\delta_S &= \frac{P_S \cdot L_S}{A_S \cdot E_S} \\
\Rightarrow \quad \delta_S &= \frac{152 \times \left(3.5 \times 10^3\right)}{1200 \times \left(200 \times 10^3\right)} \\
\Rightarrow \quad \delta_S &= 0.00222 \text{ mm}
\end{aligned}$$

The small difference in deformation is due to rounding errors

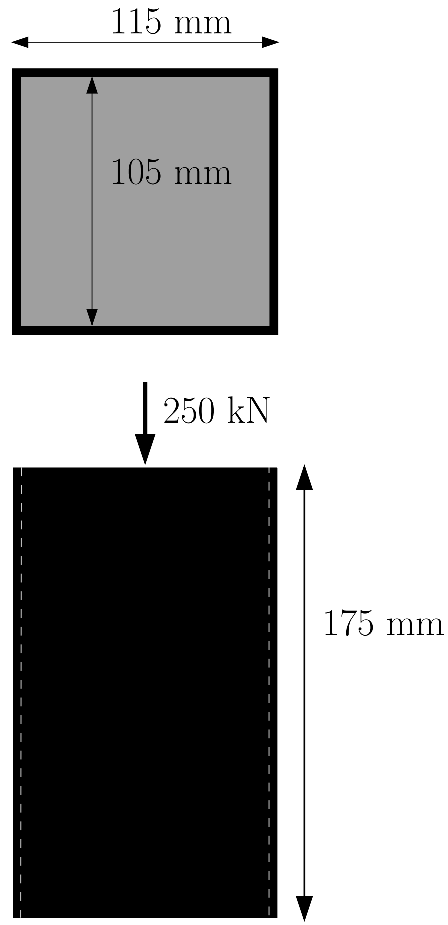
Exercise: A hollow square steel structural section has outside dimensions of 115 mm $\times$ 115 mm and inside dimensions of 105 mm $\times$ 105 mm. It is filled with concrete, as shown in plan view (upper right). The section is 3.5 m and supports a compressive load of 250 kN.

$E_S = 200$ GPa and $E_C = 20$ GPa.

Find σ_S, σ_C and δ

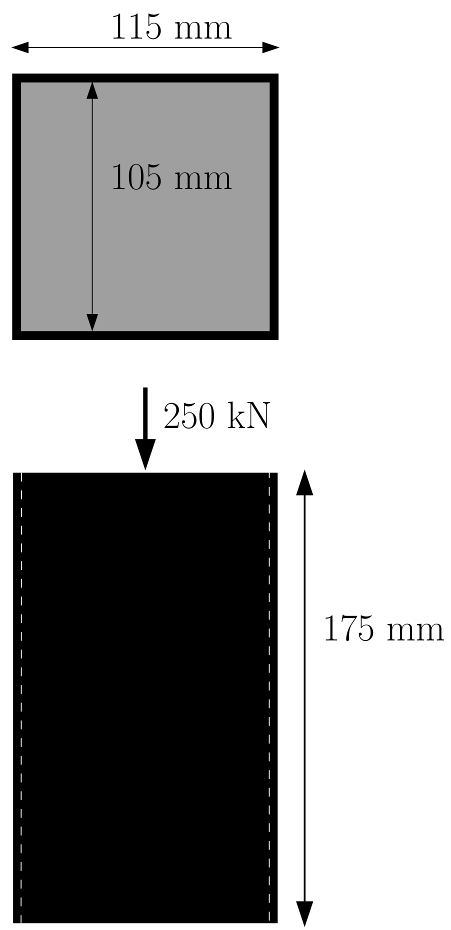


Solution: Find the areas of the steel and of the concrete:



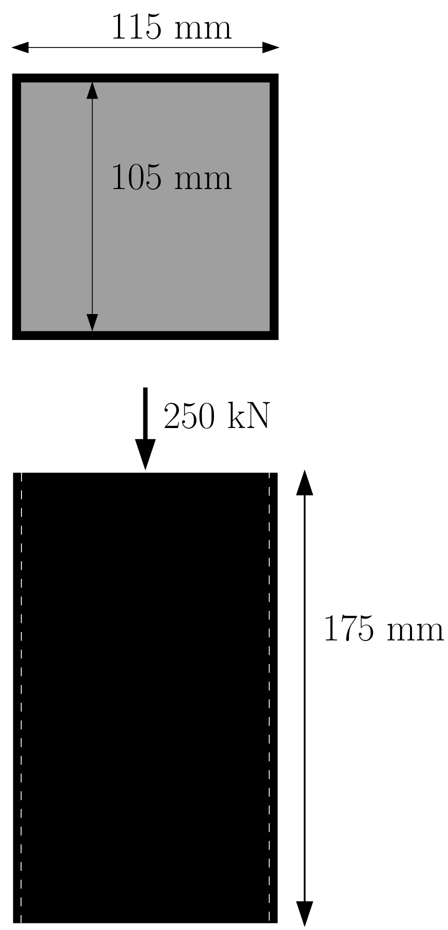
Solution: Find the areas of the steel and of the concrete:

$$A_S = (2 \times 115 \times 5) + (2 \times 105 \times 5)$$



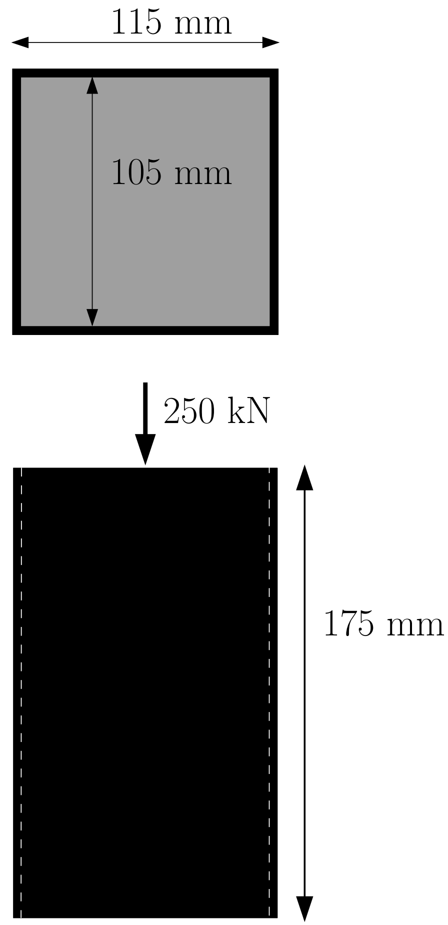
Solution: Find the areas of the steel and of the concrete:

$$
\begin{aligned}
A_S &= (2 \times 115 \times 5) + (2 \times 105 \times 5) \\
&= 2200\ \text{mm}^2
\end{aligned}
$$



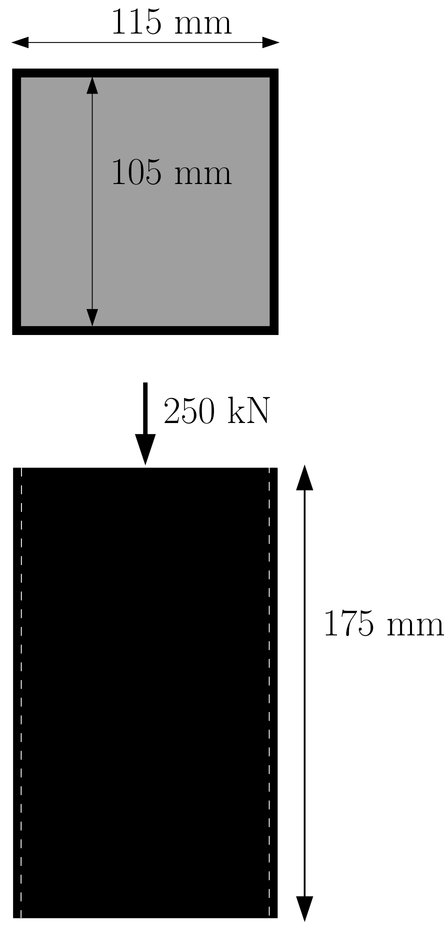
Solution: Find the areas of the steel and of the concrete:

$$A_S = (2 \times 115 \times 5) + (2 \times 105 \times 5)$$
$$= 2200 \text{ mm}^2$$
$$A_C = 105 \times 105$$



Solution: Find the areas of the steel and of the concrete:

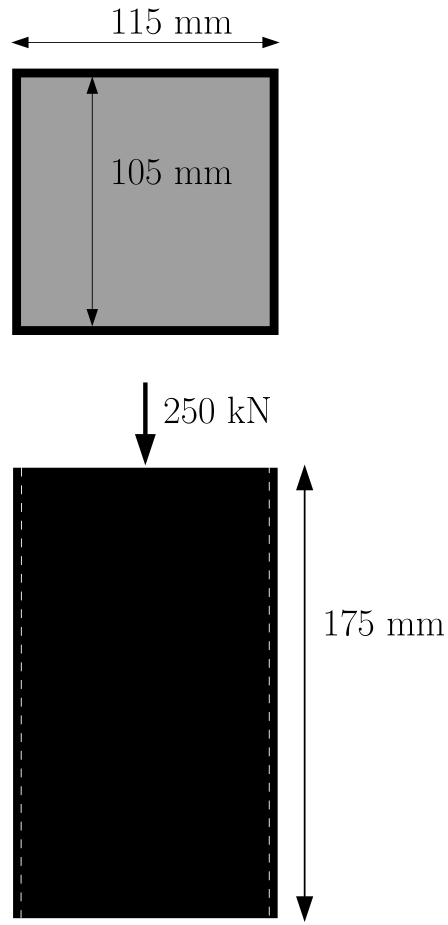
$$A_S = (2 \times 115 \times 5) + (2 \times 105 \times 5)$$
$$= 2200 \text{ mm}^2$$
$$A_C = 105 \times 105$$
$$= 11025 \text{ mm}^2$$



Solution: Find the areas of the steel and of the concrete:

$$
\begin{aligned}
A_S &= (2 \times 115 \times 5) + (2 \times 105 \times 5) \\
&= 2200\ \text{mm}^2 \\
A_C &= 105 \times 105 \\
&= 11025\ \text{mm}^2
\end{aligned}
$$

Let P_S be the reaction force of the steel and P_C the reaction force of the concrete.

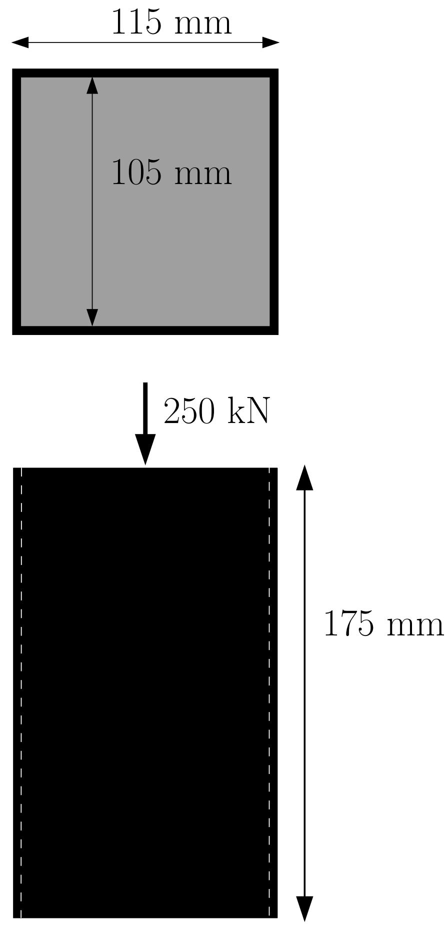


Solution: Find the areas of the steel and of the concrete:

$$A_S = (2 \times 115 \times 5) + (2 \times 105 \times 5)$$
$$= 2200 \text{ mm}^2$$
$$A_C = 105 \times 105$$
$$= 11025 \text{ mm}^2$$

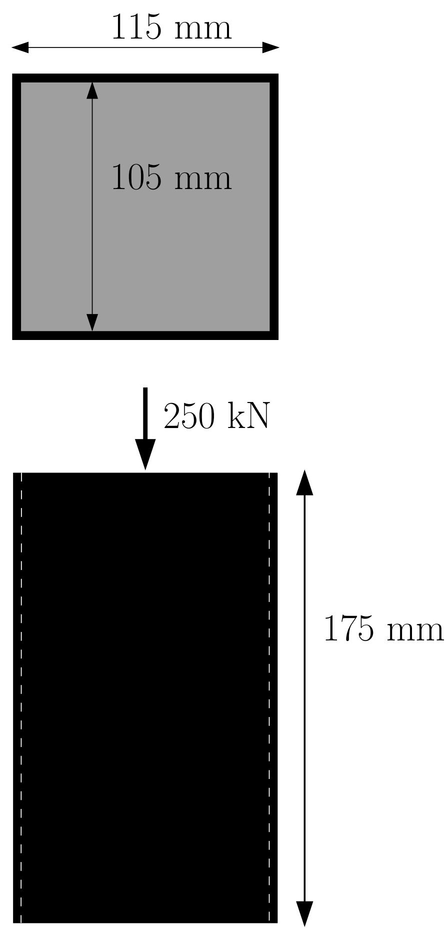
Let P_S be the reaction force of the steel and P_C the reaction force of the concrete. Then,

$$\Sigma F_y = P_S + P_C - 250 = 0$$



Solution: Find the areas of the steel and of the concrete:

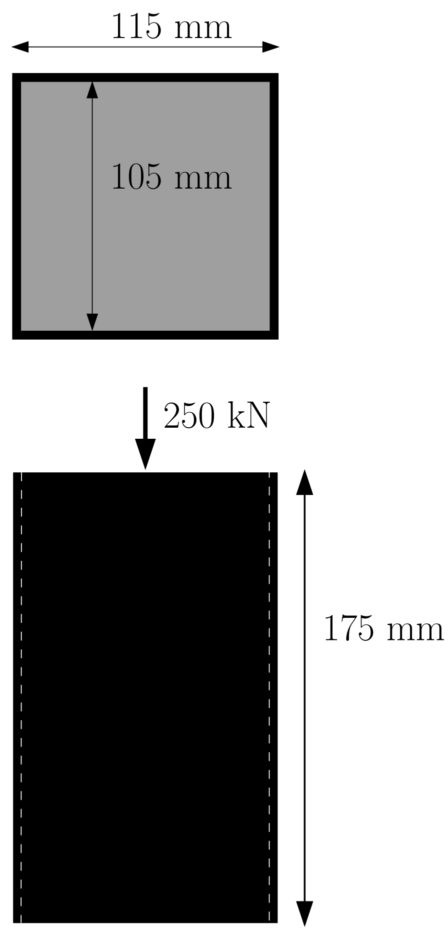
$$
\begin{aligned}
A_S &= (2 \times 115 \times 5) + (2 \times 105 \times 5) \\
&= 2200 \text{ mm}^2 \\
A_C &= 105 \times 105 \\
&= 11025 \text{ mm}^2
\end{aligned}
$$



Let P_S be the reaction force of the steel and P_C the reaction force of the concrete. Then,

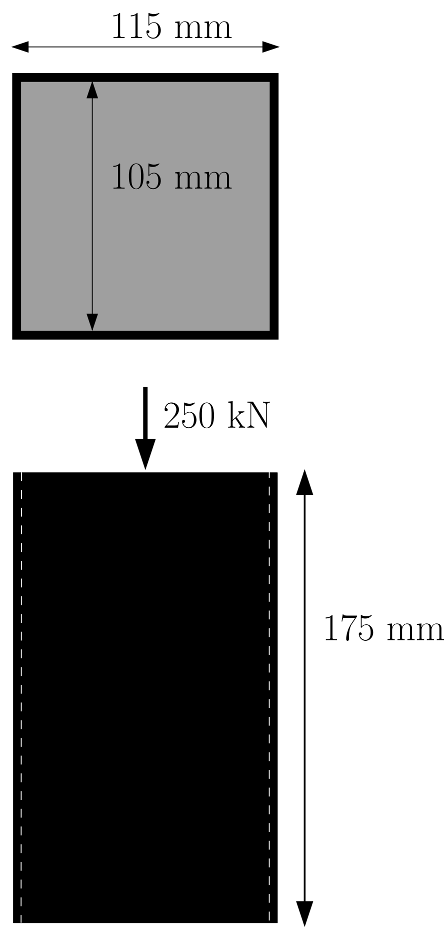
$$
\begin{aligned}
\Sigma F_y = P_S + P_C - 250 &= 0 \\
P_S + P_C &= 250 \text{ kN}
\end{aligned}
$$

Solution: The steel casing and the concrete both deform by the same amount



Solution: The steel casing and the concrete both deform by the same amount

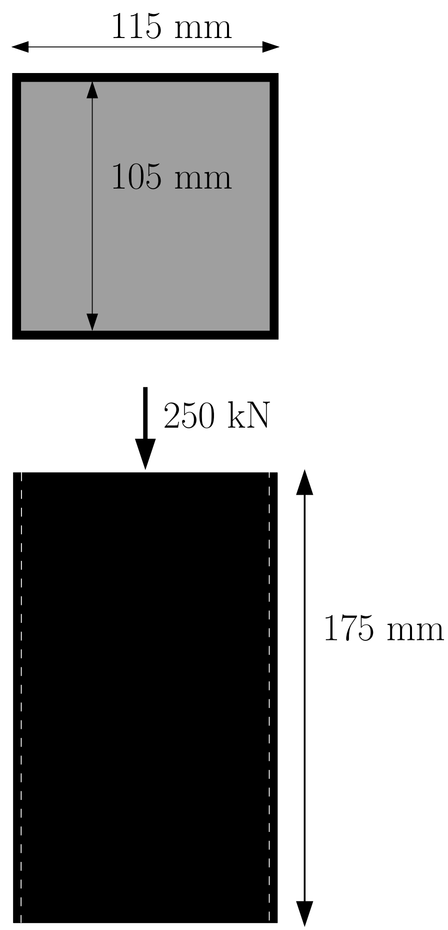
$$\frac{P_C \cdot L_C}{A_C \cdot E_C} = \frac{P_S \cdot L_S}{A_S \cdot E_S}$$



Solution: The steel casing and the concrete both deform by the same amount

$$\frac{P_C \cdot L_C}{A_C \cdot E_C} = \frac{P_S \cdot L_S}{A_S \cdot E_S}$$

$$\Rightarrow \quad \frac{\left(P_C \times 10^3\right) \times 175}{11025 \times (200 \times 10^3)} = \frac{\left(P_S \times 10^3\right) \times 175}{2200 \times (200 \times 10^3)}$$



Solution: The steel casing and the concrete both deform by the same amount

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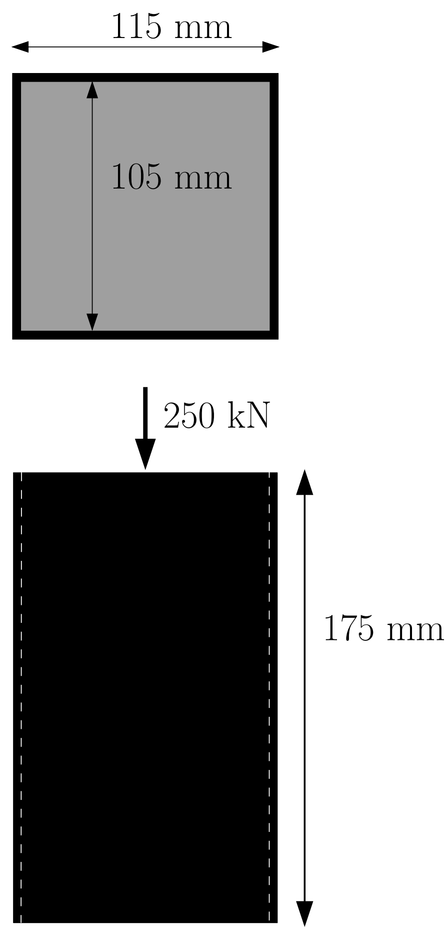
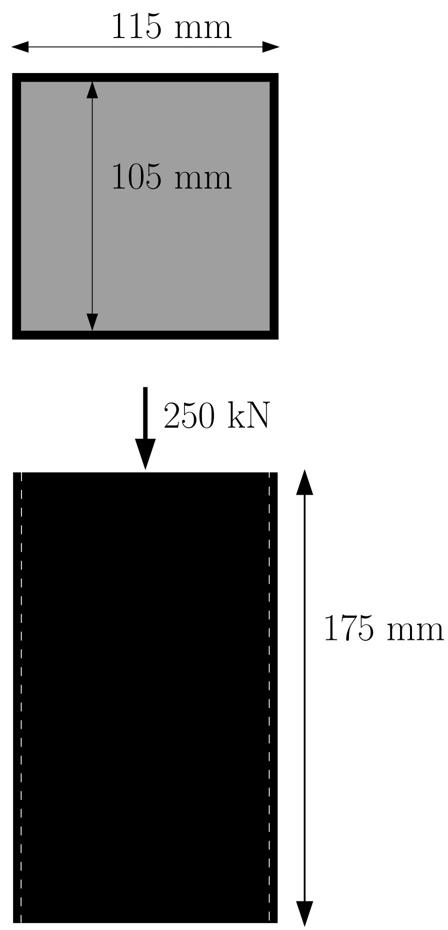
$$\Rightarrow \quad \frac{\left(P_C \times 10^3\right) \times 175}{11025 \times (200 \times 10^3)} = \frac{\left(P_S \times 10^3\right) \times 175}{2200 \times (200 \times 10^3)}$$

$$\Rightarrow \quad \frac{P_C}{11025 \times 20} = \frac{P_S}{2200 \times 200}$$



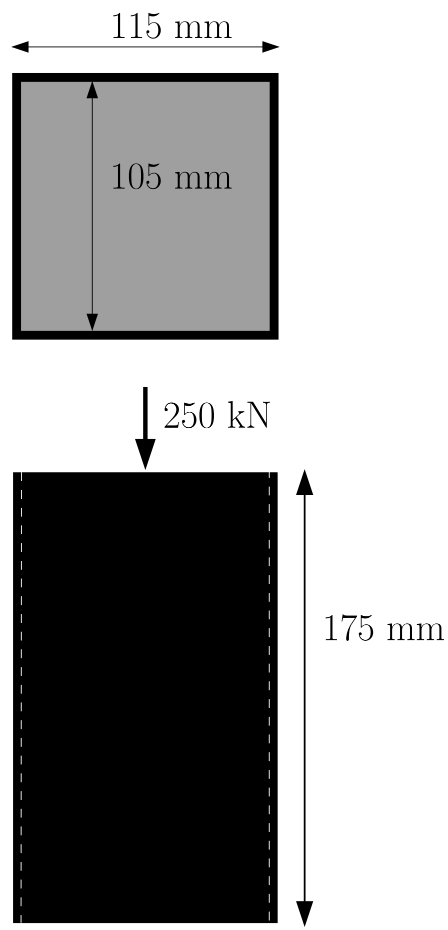
Solution: The steel casing and the concrete both deform by the same amount

$$
\begin{aligned}
\frac{P_C \cdot L_C}{A_C \cdot E_C} &= \frac{P_S \cdot L_S}{A_S \cdot E_S} \\
\Rightarrow \quad \frac{\left(P_C \times 10^3\right) \times 175}{11025 \times (200 \times 10^3)} &= \frac{\left(P_S \times 10^3\right) \times 175}{2200 \times (200 \times 10^3)} \\
\Rightarrow \quad \frac{P_C}{11025 \times 20} &= \frac{P_S}{2200 \times 200} \\
\Rightarrow \quad P_C &= \frac{11025 \times 20}{2200 \times 200} \cdot P_S
\end{aligned}
$$



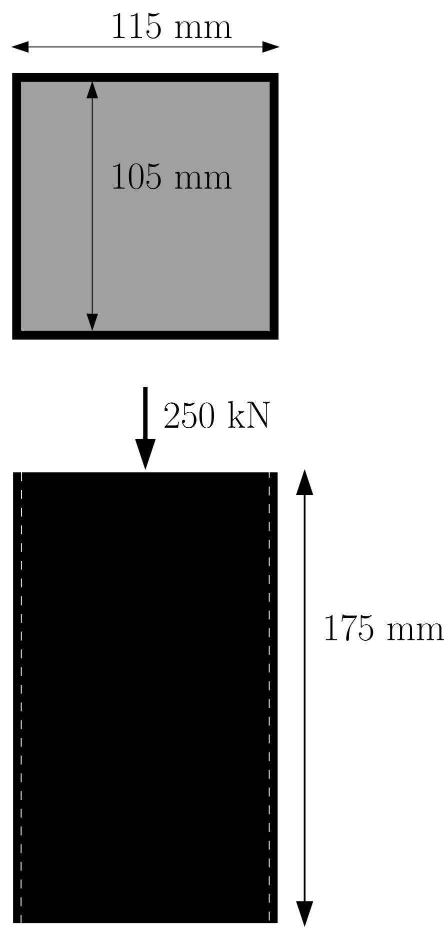
Solution: The steel casing and the concrete both deform by the same amount

$$
\begin{aligned}
\frac{P_C \cdot L_C}{A_C \cdot E_C} &= \frac{P_S \cdot L_S}{A_S \cdot E_S} \\
\Rightarrow \quad \frac{\left(P_C \times 10^3\right) \times 175}{11025 \times (200 \times 10^3)} &= \frac{\left(P_S \times 10^3\right) \times 175}{2200 \times (200 \times 10^3)} \\
\Rightarrow \quad \frac{P_C}{11025 \times 20} &= \frac{P_S}{2200 \times 200} \\
\Rightarrow \quad P_C &= \frac{11025 \times 20}{2200 \times 200} \cdot P_S \\
\Rightarrow \quad P_C &= 0.5011 P_S
\end{aligned}
$$



Solution: The steel casing and the concrete both deform by the same amount

$$
\begin{aligned}
\frac{P_C \cdot L_C}{A_C \cdot E_C} &= \frac{P_S \cdot L_S}{A_S \cdot E_S} \\
\Rightarrow \quad \frac{\left(P_C \times 10^3\right) \times 175}{11025 \times (200 \times 10^3)} &= \frac{\left(P_S \times 10^3\right) \times 175}{2200 \times (200 \times 10^3)} \\
\Rightarrow \quad \frac{P_C}{11025 \times 20} &= \frac{P_S}{2200 \times 200} \\
\Rightarrow \quad P_C &= \frac{11025 \times 20}{2200 \times 200} \cdot P_S \\
\Rightarrow \quad P_C &= 0.5011 P_S
\end{aligned}
$$



$\Sigma F_y = 0$ so

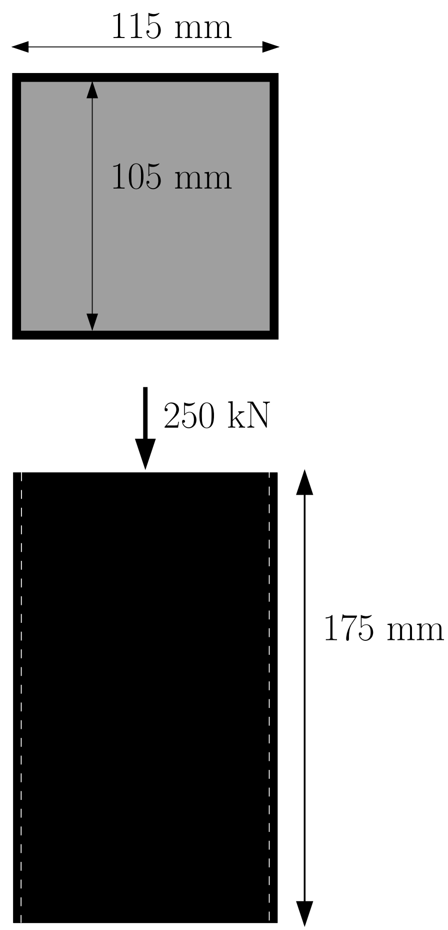
$$P_C + P_S - 250 = 0$$

Solution: The steel casing and the concrete both deform by the same amount

$$
\begin{aligned}
\frac{P_C \cdot L_C}{A_C \cdot E_C} &= \frac{P_S \cdot L_S}{A_S \cdot E_S} \\
\Rightarrow \quad \frac{\left(P_C \times 10^3\right) \times 175}{11025 \times (200 \times 10^3)} &= \frac{\left(P_S \times 10^3\right) \times 175}{2200 \times (200 \times 10^3)} \\
\Rightarrow \quad \frac{P_C}{11025 \times 20} &= \frac{P_S}{2200 \times 200} \\
\Rightarrow \quad P_C &= \frac{11025 \times 20}{2200 \times 200} \cdot P_S \\
\Rightarrow \quad P_C &= 0.5011 P_S
\end{aligned}
$$

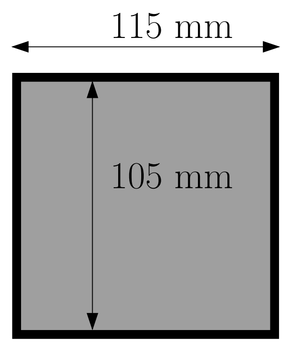
$\Sigma F_y = 0$ so

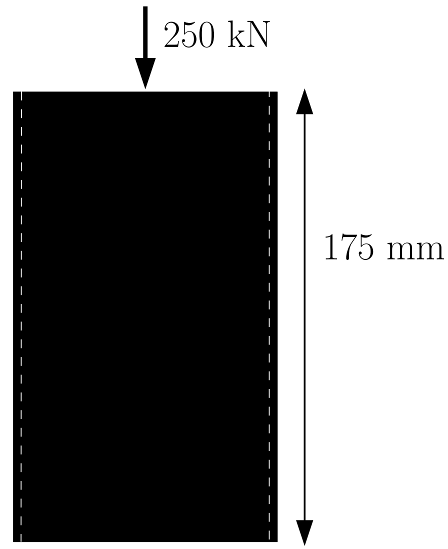
$$
\begin{aligned}
P_C + P_S - 250 &= 0 \\
\Rightarrow \quad P_C &= 250 - P_S
\end{aligned}
$$



Solution: The steel casing and the concrete both deform by the same amount

$$\begin{aligned}
\frac{P_C \cdot L_C}{A_C \cdot E_C} &= \frac{P_S \cdot L_S}{A_S \cdot E_S} \\
\Rightarrow \quad \frac{\left(P_C \times 10^3\right) \times 175}{11025 \times (200 \times 10^3)} &= \frac{\left(P_S \times 10^3\right) \times 175}{2200 \times (200 \times 10^3)} \\
\Rightarrow \quad \frac{P_C}{11025 \times 20} &= \frac{P_S}{2200 \times 200} \\
\Rightarrow \quad P_C &= \frac{11025 \times 20}{2200 \times 200} \cdot P_S \\
\Rightarrow \quad P_C &= 0.5011 P_S
\end{aligned}$$





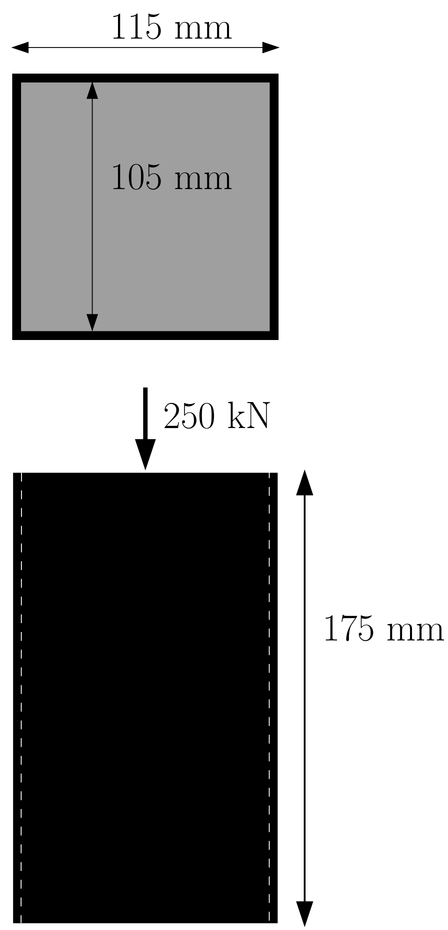
$\Sigma F_y = 0$ so

$$\begin{aligned}
P_C + P_S - 250 &= 0 \\
\Rightarrow \quad P_C &= 250 - P_S
\end{aligned}$$

Now we have two equations for the two unknowns, P_C and P_S

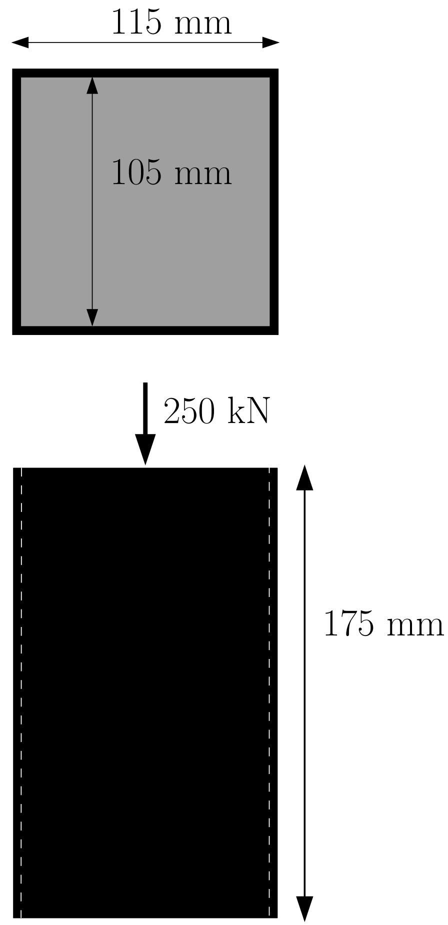
Solution:

$$
\begin{aligned}
P_C &= 0.5011 P_S \\
P_C &= 250 - P_S
\end{aligned}
$$



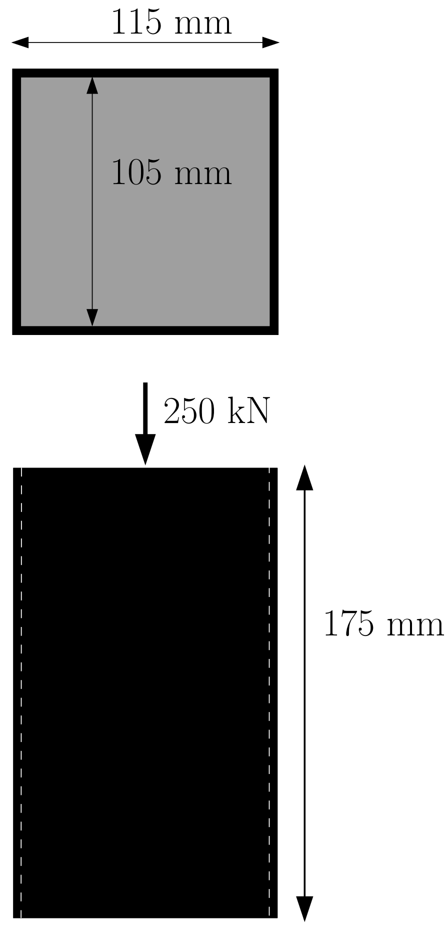
Solution:

$$
\begin{aligned}
P_C &= 0.5011P_S \\
P_C &= 250 - P_S \\
\Rightarrow \quad 250 - P_S &= 0.5011P_S
\end{aligned}
$$



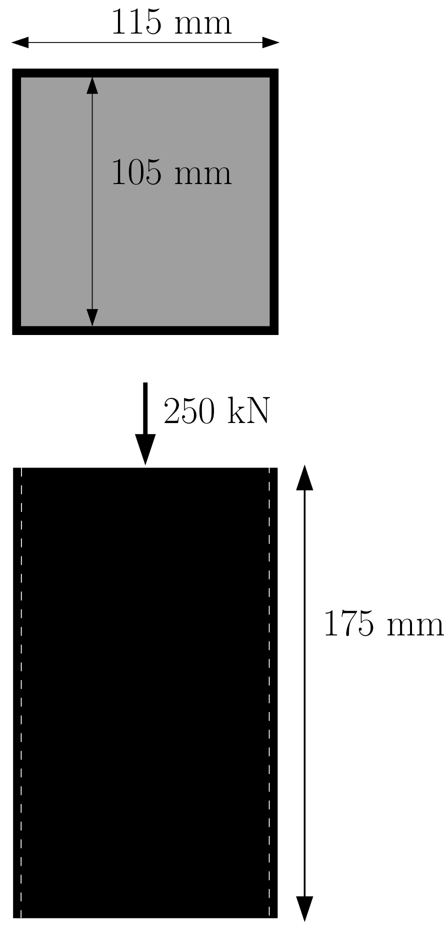
Solution:

$$
\begin{aligned}
P_C &= 0.5011 P_S \\
P_C &= 250 - P_S \\
\Rightarrow \quad 250 - P_S &= 0.5011 P_S \\
\Rightarrow \quad P_S &= \tfrac{250}{1+0.5011}
\end{aligned}
$$



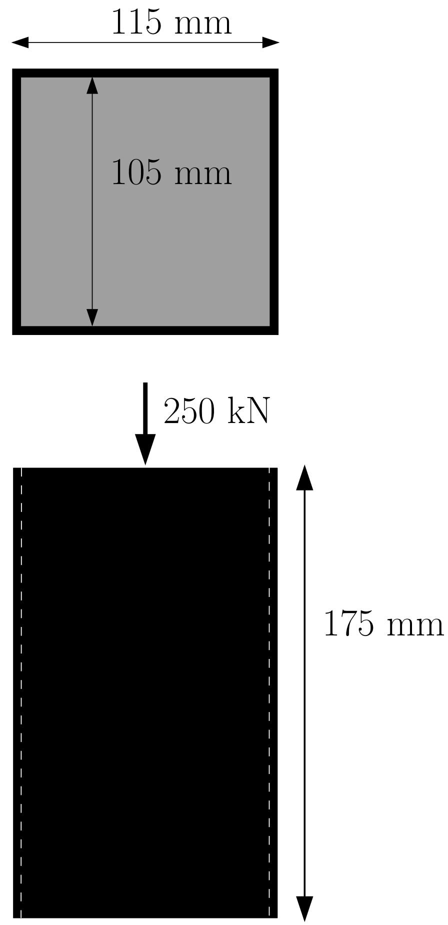
Solution:

$$
\begin{aligned}
P_C &= 0.5011 P_S \\
P_C &= 250 - P_S \\
\Rightarrow \quad 250 - P_S &= 0.5011 P_S \\
\Rightarrow \quad P_S &= \tfrac{250}{1+0.5011} \\
\Rightarrow \quad P_S &= 166.5 \text{ kN}
\end{aligned}
$$



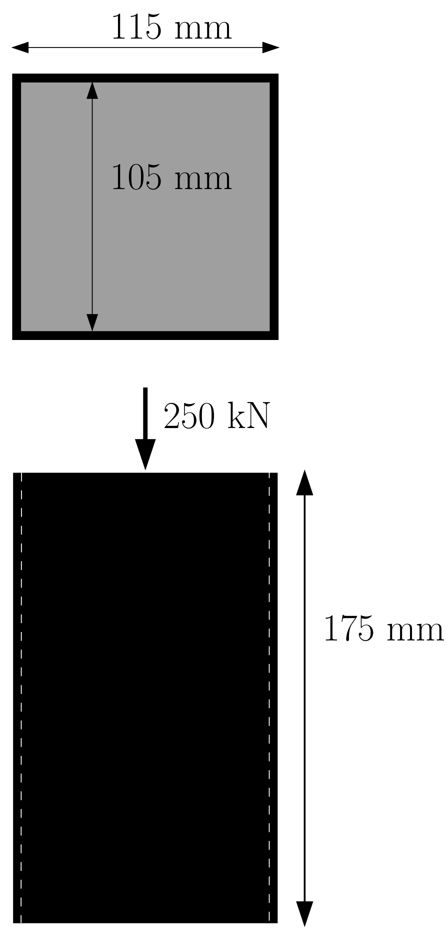
Solution:

$$
\begin{aligned}
P_C &= 0.5011 P_S \\
P_C &= 250 - P_S \\
\Rightarrow \quad 250 - P_S &= 0.5011 P_S \\
\Rightarrow \quad P_S &= \frac{250}{1+0.5011} \\
\Rightarrow \quad P_S &= 166.5 \text{ kN} \\
\Rightarrow \quad P_C &= 83.5 \text{ kN}
\end{aligned}
$$



Solution:

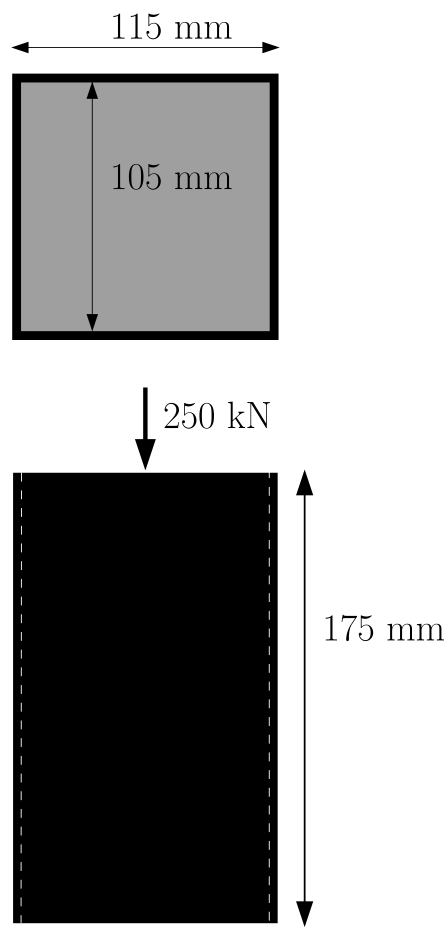
$$
\begin{aligned}
& P_C &=& \; 0.5011 P_S \\
& P_C &=& \; 250 - P_S \\
\Rightarrow \quad & 250 - P_S &=& \; 0.5011 P_S \\
\Rightarrow \quad & P_S &=& \; \frac{250}{1+0.5011} \\
\Rightarrow \quad & P_S &=& \; 166.5 \text{ kN} \\
\Rightarrow \quad & P_C &=& \; 83.5 \text{ kN}
\end{aligned}
$$



Now, find the stress in the steel:

Solution:

$$\begin{aligned}
P_C &= 0.5011 P_S \\
P_C &= 250 - P_S \\
\Rightarrow \quad 250 - P_S &= 0.5011 P_S \\
\Rightarrow \quad P_S &= \frac{250}{1+0.5011} \\
\Rightarrow \quad P_S &= 166.5 \text{ kN} \\
\Rightarrow \quad P_C &= 83.5 \text{ kN}
\end{aligned}$$

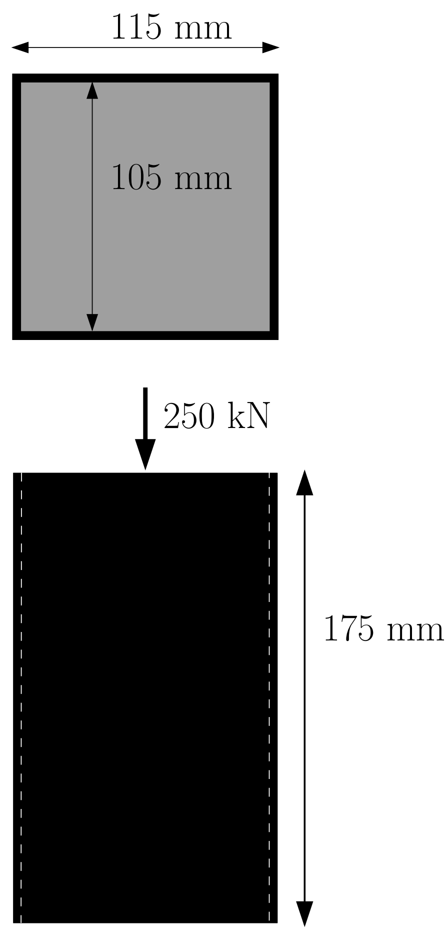


Now, find the stress in the steel:

$$\sigma_S = \frac{P_S}{A_S}$$

Solution:

$$
\begin{aligned}
P_C &= 0.5011 P_S \\
P_C &= 250 - P_S \\
\Rightarrow \quad 250 - P_S &= 0.5011 P_S \\
\Rightarrow \quad P_S &= \frac{250}{1+0.5011} \\
\Rightarrow \quad P_S &= 166.5 \text{ kN} \\
\Rightarrow \quad P_C &= 83.5 \text{ kN}
\end{aligned}
$$

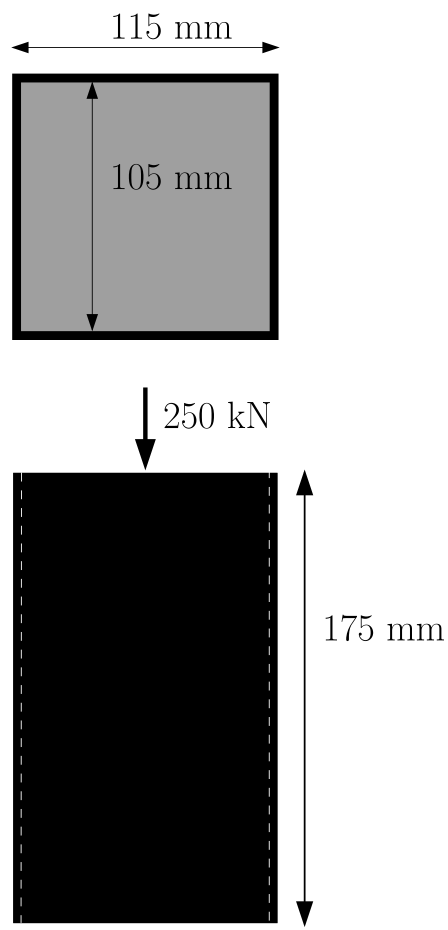


Now, find the stress in the steel:

$$
\begin{aligned}
\sigma_S &= \frac{P_S}{A_S} \\
\Rightarrow \quad \sigma_S &= \frac{166.5 \times 10^3}{2200}
\end{aligned}
$$

Solution:

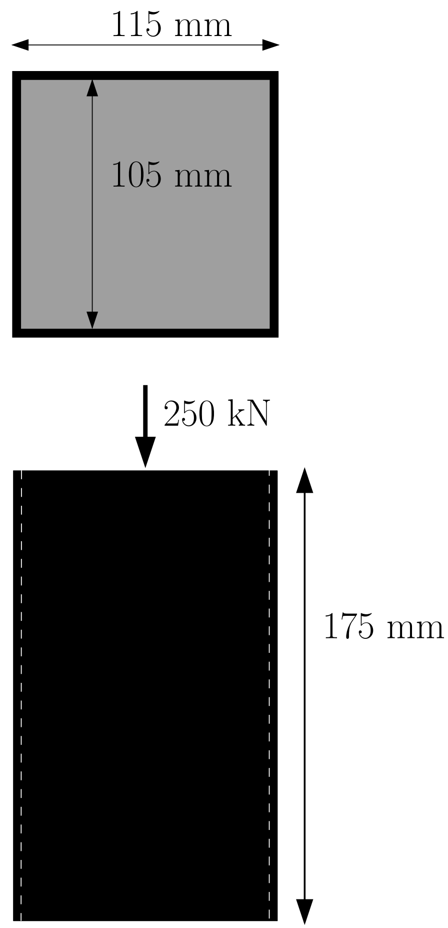
$$
\begin{aligned}
P_C &= 0.5011 P_S \\
P_C &= 250 - P_S \\
\Rightarrow \quad 250 - P_S &= 0.5011 P_S \\
\Rightarrow \quad P_S &= \frac{250}{1+0.5011} \\
\Rightarrow \quad P_S &= 166.5 \text{ kN} \\
\Rightarrow \quad P_C &= 83.5 \text{ kN}
\end{aligned}
$$



Now, find the stress in the steel:

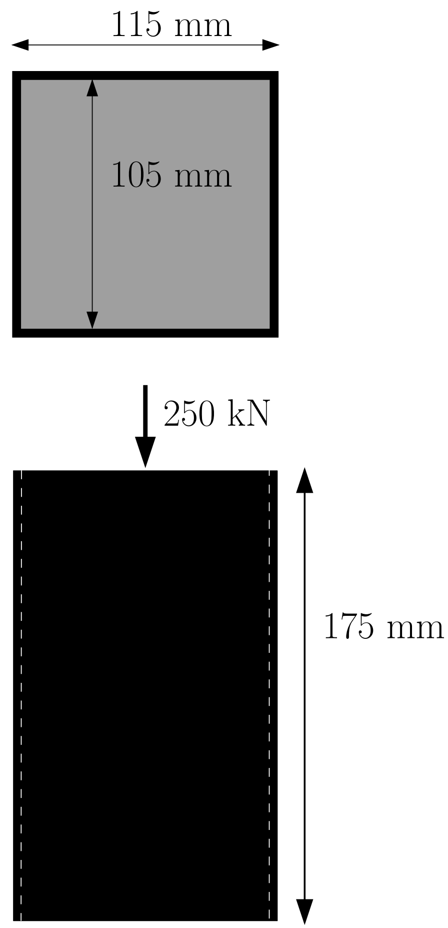
$$
\begin{aligned}
\sigma_S &= \frac{P_S}{A_S} \\
\Rightarrow \quad \sigma_S &= \frac{166.5 \times 10^3}{2200} \\
\Rightarrow \quad \sigma_S &= 75.7 \text{ MPa}
\end{aligned}
$$

Solution: Now, find the stress in the concrete:



Solution: Now, find the stress in the concrete:

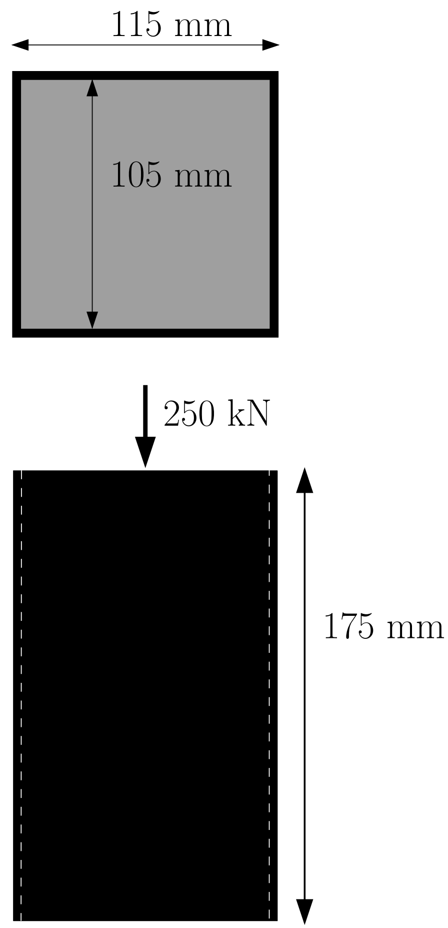
$$\sigma_C = \frac{P_C}{A_C}$$



Solution: Now, find the stress in the concrete:

$$\sigma_C = \frac{P_C}{A_C}$$

$$\Rightarrow \sigma_C = \frac{83.5 \times 10^3}{11025}$$

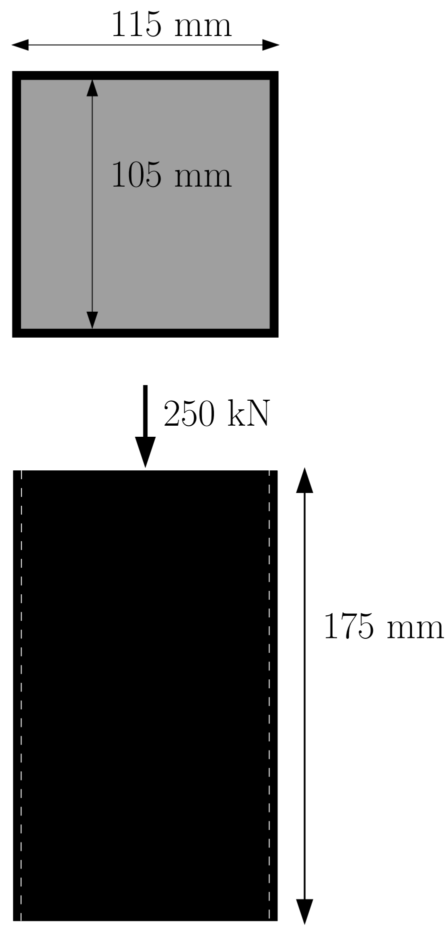


Solution: Now, find the stress in the concrete:

$$\sigma_C = \frac{P_C}{A_C}$$

$$\Rightarrow \sigma_C = \frac{83.5\times 10^3}{11025}$$

$$\Rightarrow \sigma_C = 7.57 \text{ MPa}$$



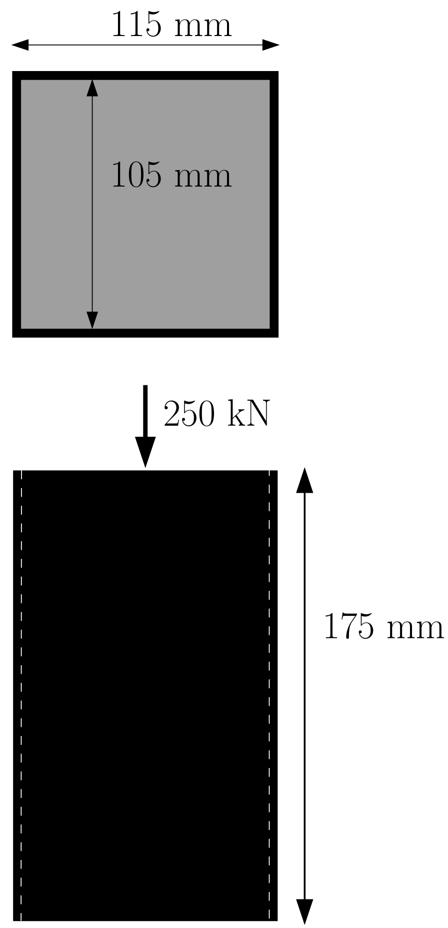
Solution: Now, find the stress in the concrete:

$$\sigma_C = \frac{P_C}{A_C}$$

$$\Rightarrow \sigma_C = \frac{83.5\times10^3}{11025}$$

$$\Rightarrow \sigma_C = 7.57 \text{ MPa}$$

Now, find the deformation:



Solution: Now, find the stress in the concrete:

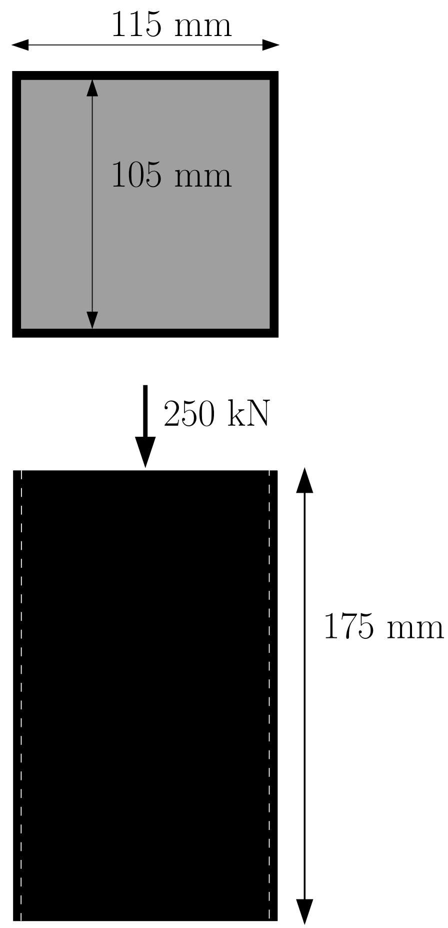
$$\sigma_C = \frac{P_C}{A_C}$$

$$\Rightarrow \sigma_C = \frac{83.5 \times 10^3}{11025}$$

$$\Rightarrow \sigma_C = 7.57 \text{ MPa}$$

Now, find the deformation:

$$\delta = \frac{P_C \cdot L_C}{A_C \cdot E_C}$$



Solution: Now, find the stress in the concrete:

$$\sigma_C = \frac{P_C}{A_C}$$

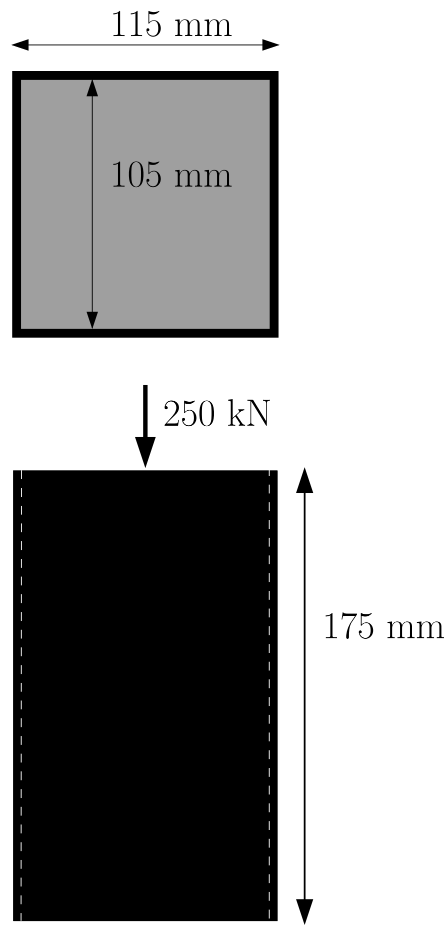
$$\Rightarrow \sigma_C = \frac{83.5\times10^3}{11025}$$

$$\Rightarrow \sigma_C = 7.57 \text{ MPa}$$

Now, find the deformation:

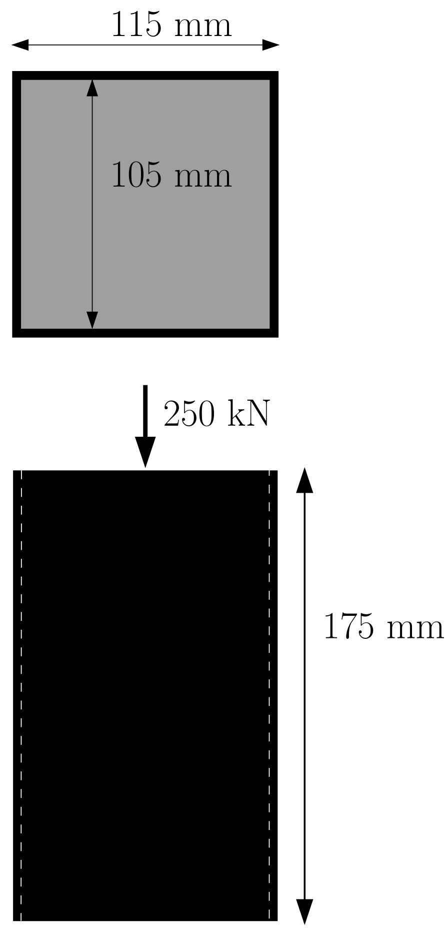
$$\delta = \frac{P_C \cdot L_C}{A_C \cdot E_C}$$

$$\Rightarrow \delta = \frac{\left(83.5\times10^3\right)\times175}{11025\times(20\times10^3)}$$



Solution: Now, find the stress in the concrete:

$$
\begin{aligned}
\sigma_C &= \frac{P_C}{A_C} \\
\Rightarrow \quad \sigma_C &= \frac{83.5 \times 10^3}{11025} \\
\Rightarrow \quad \sigma_C &= 7.57 \text{ MPa}
\end{aligned}
$$



Now, find the deformation:

$$
\begin{aligned}
\delta &= \frac{P_C \cdot L_C}{A_C \cdot E_C} \\
\Rightarrow \quad \delta &= \frac{\left(83.5 \times 10^3\right) \times 175}{11025 \times (20 \times 10^3)} \\
\Rightarrow \quad \delta &= 0.0663 \text{ mm}
\end{aligned}
$$

Created by Dave Morgan using $\LaTeX\,2_\varepsilon$ and *Prosper* on January 26, 2006