Stress-Strain Relationship

(Strength of Materials)

Dave Morgan

<david.morgan@sait.ca>

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- Provides information about how the material responds to an applied tensile force
- We can find:
  - The proportional limit
  - E, the modulus of elasticity (Young's Modulus)
  - The yield strength
  - The ultimate strength

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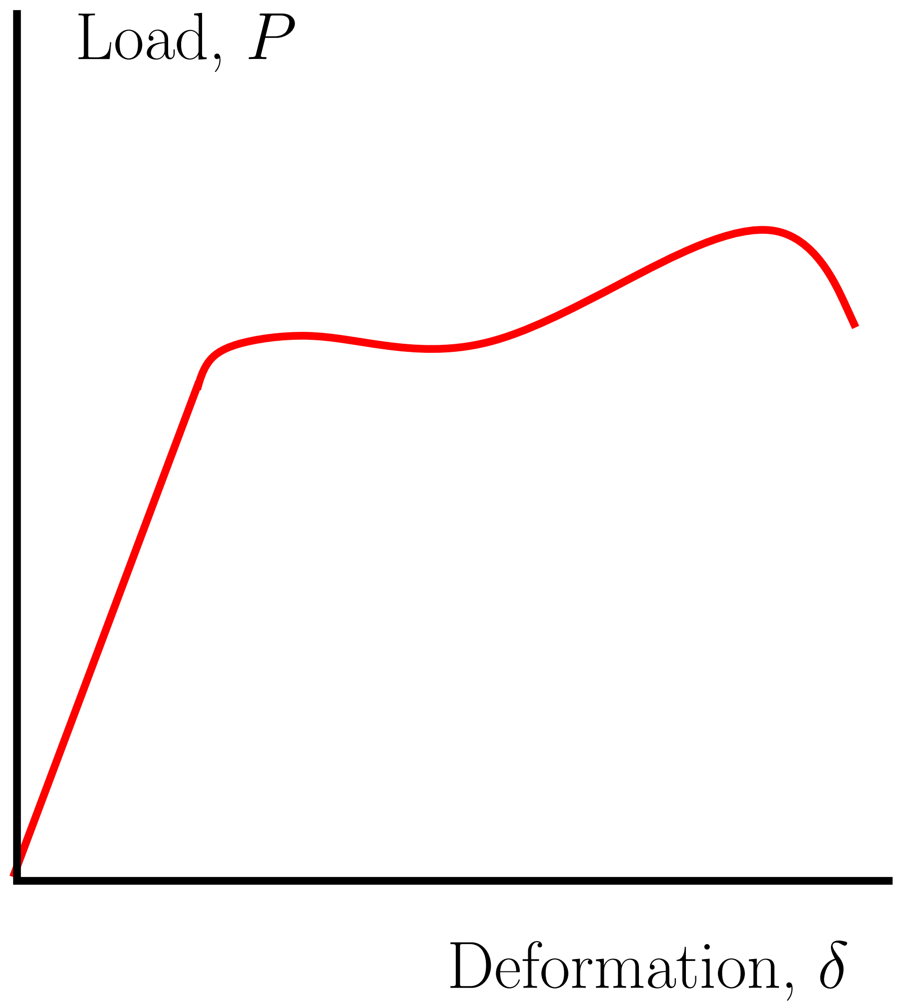
- For a range of loads, load and the corresponding deformation are recorded
- Load, P, and deformation, δ, are plotted on a graph


Load, P
Deformation, δ

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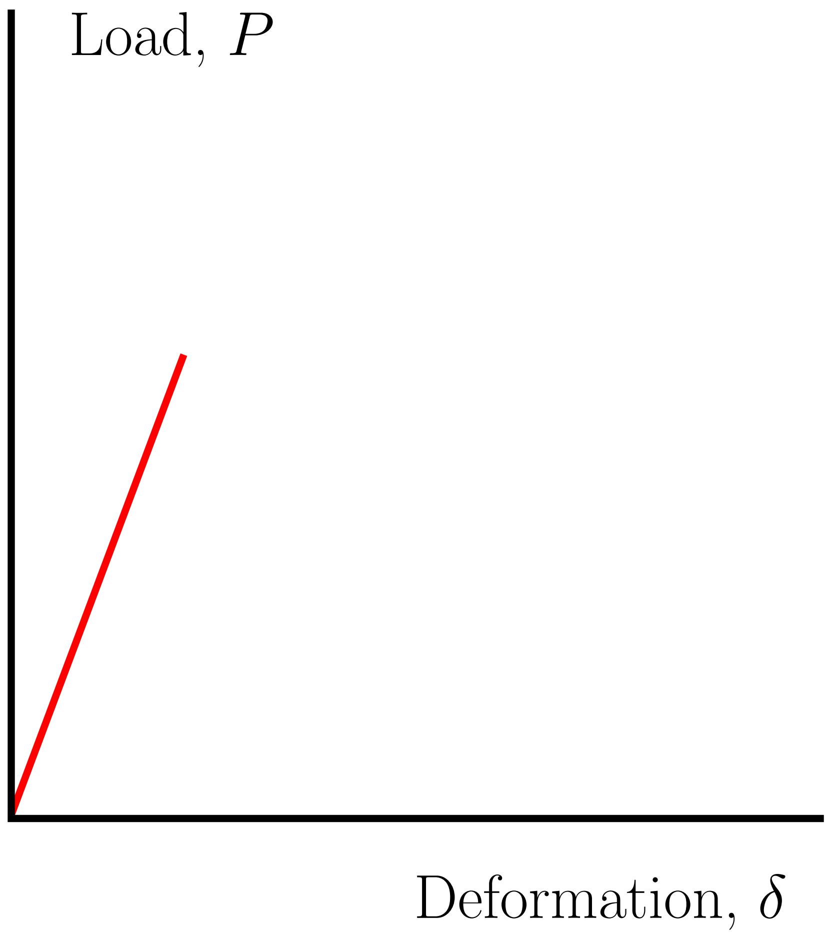
  - We can find E from L, A and the slope of the plotted graph (which is $\frac{P}{\delta}$)

Up to the proportional limit for the material, the graph is a straight line



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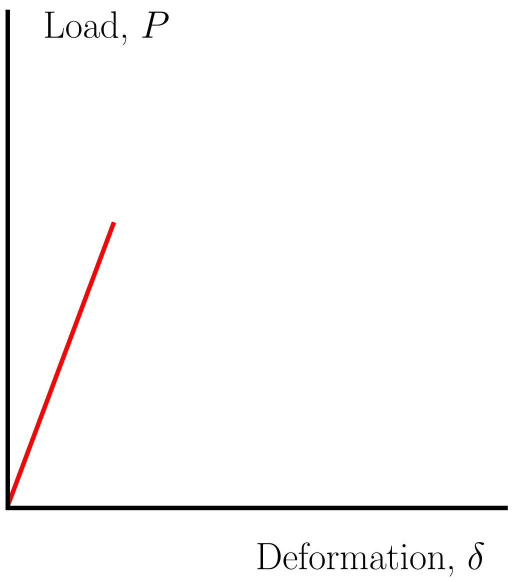

- Load is proportional to deformation
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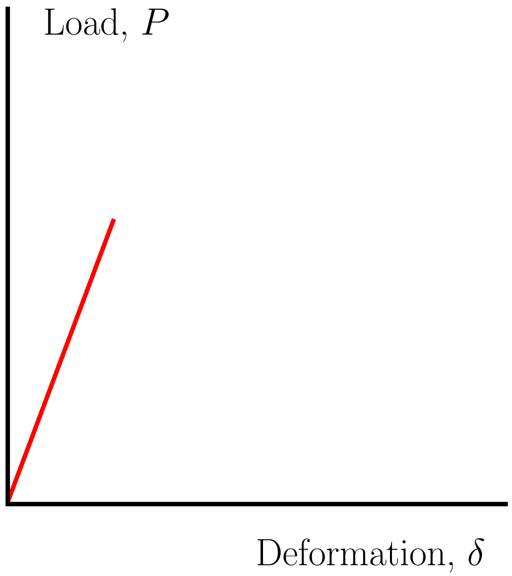
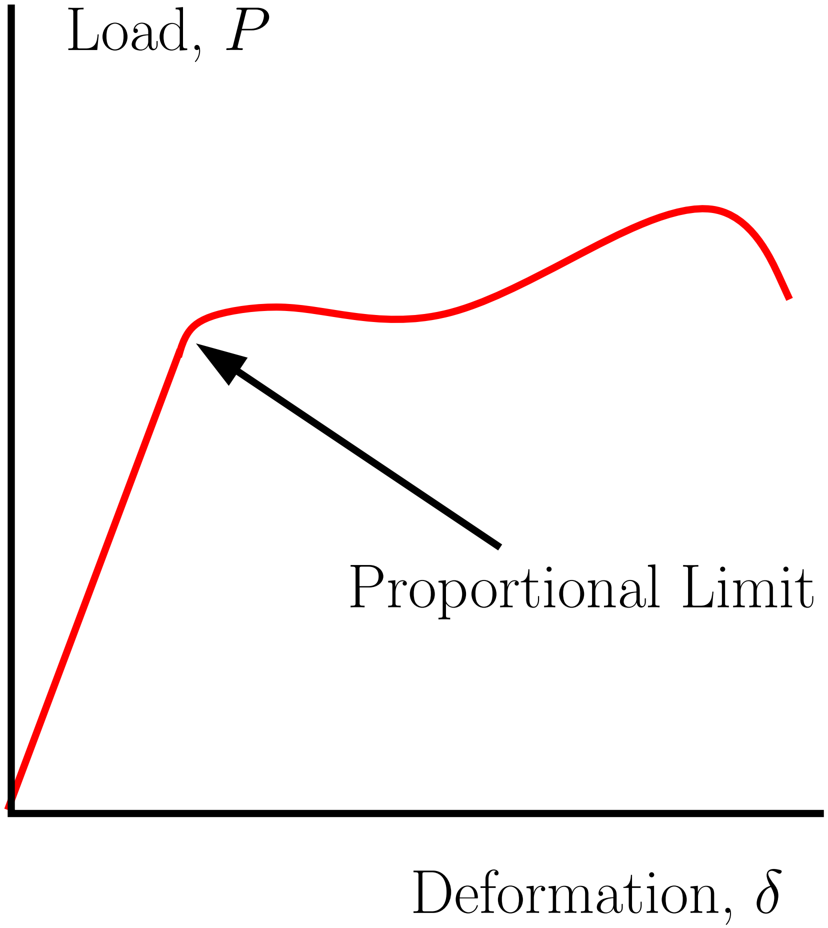
- Material behaves *elastically*

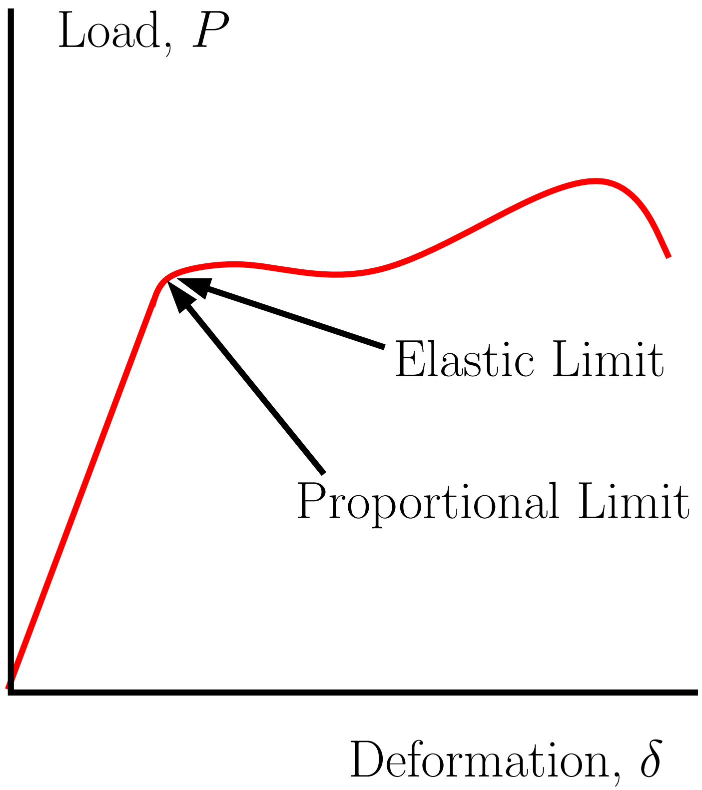


- Load is proportional to deformation
- Stress is proportional to strain
- Material behaves *elastically*
    - There is no permanent change to the material; when the load is removed, the material resumes its original shape

At the proportional limit, the graph changes from a straight line



The *elastic limit* is the point after which the sample will not return to its original shape when the load is released.



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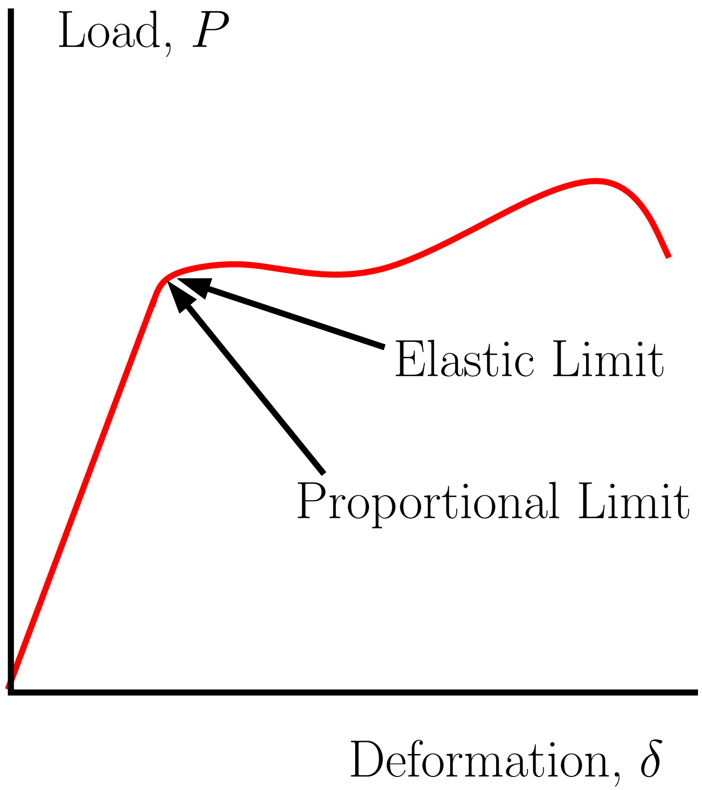

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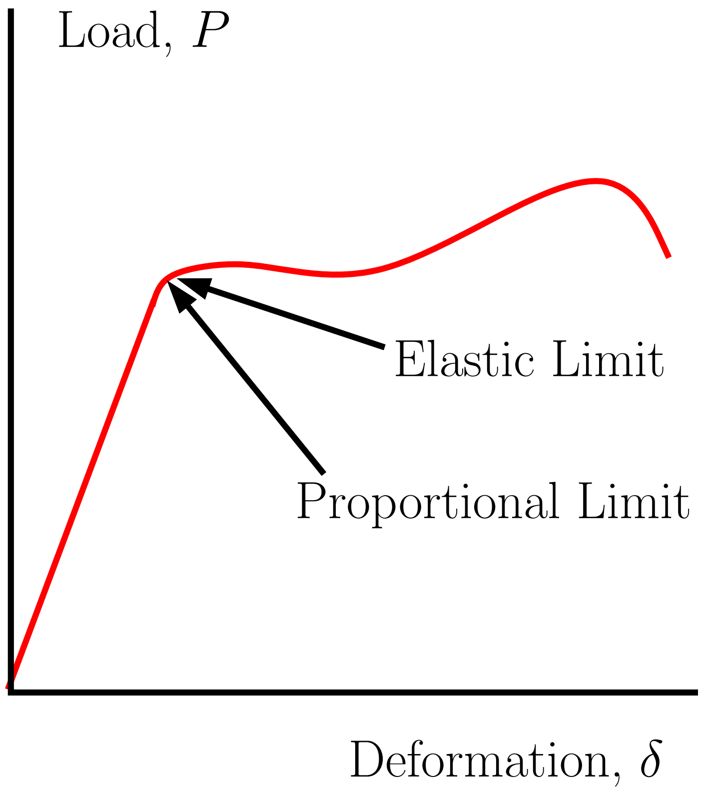
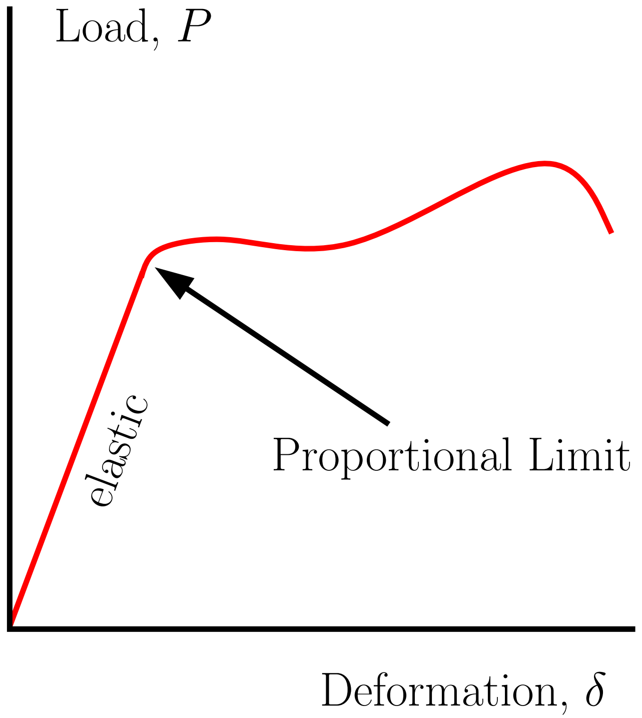


- The proportional limit and the elastic limit are very close. For most purposes, we may consider them to be the same point.
- There is permanent change to the structure of the material.

There may be a region of increased deformation without increased load



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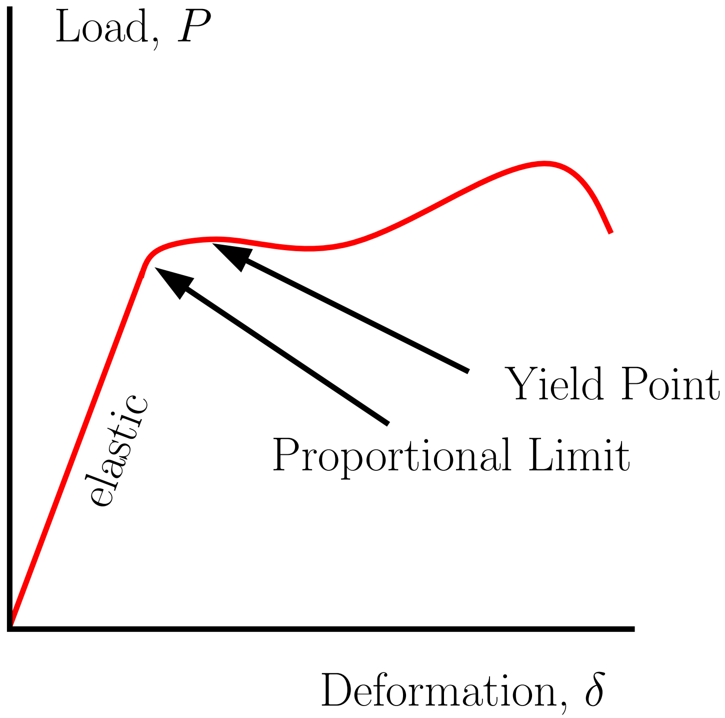

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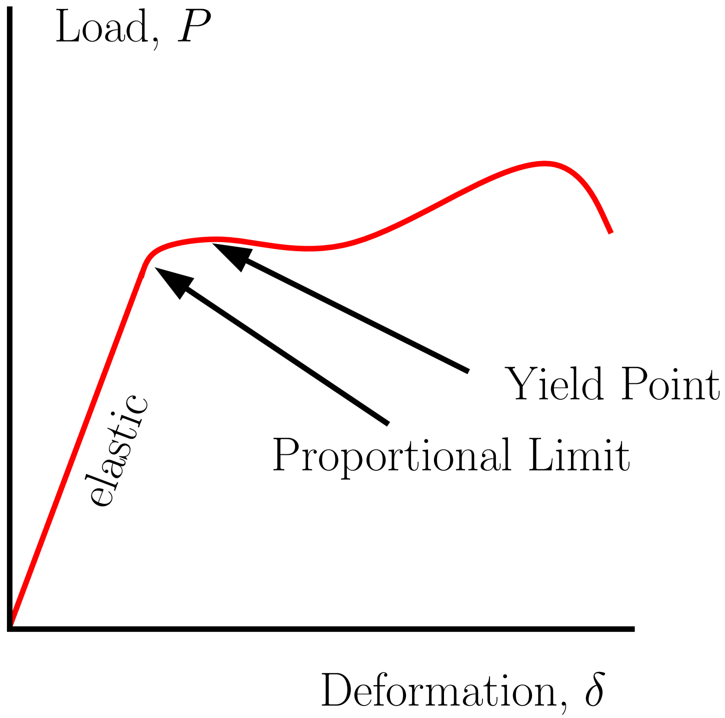

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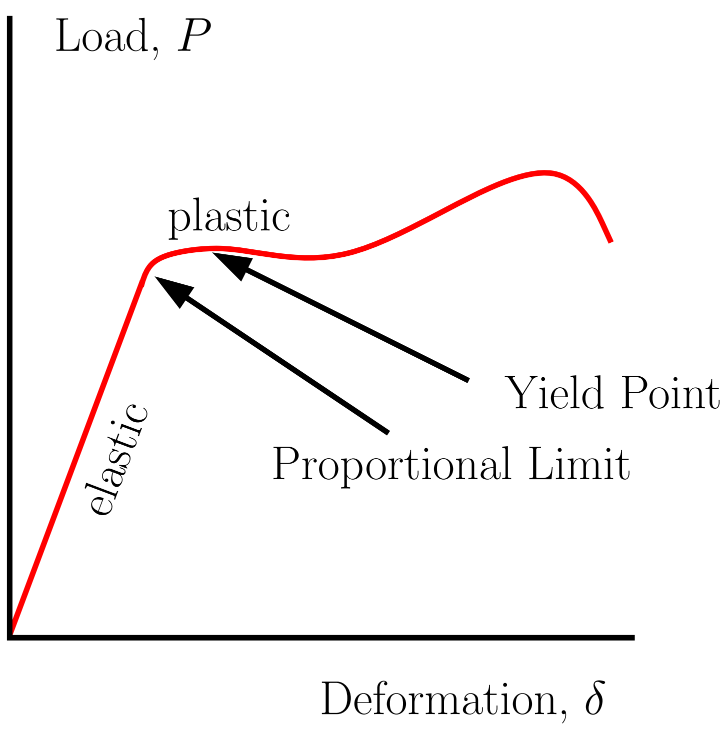
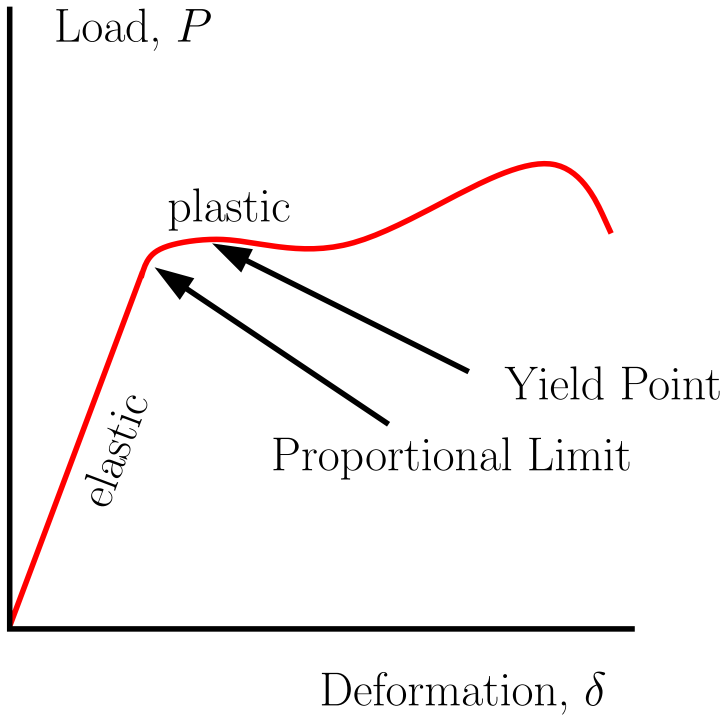


- This point is known as the *yield point*
- The stress at this point is called the *yield strength*.
- The material behaves *plastically* (when the load is removed, the sample does not return to its original shape)

After the yield point, there may be a region of where increased load is necessary for increased deformation



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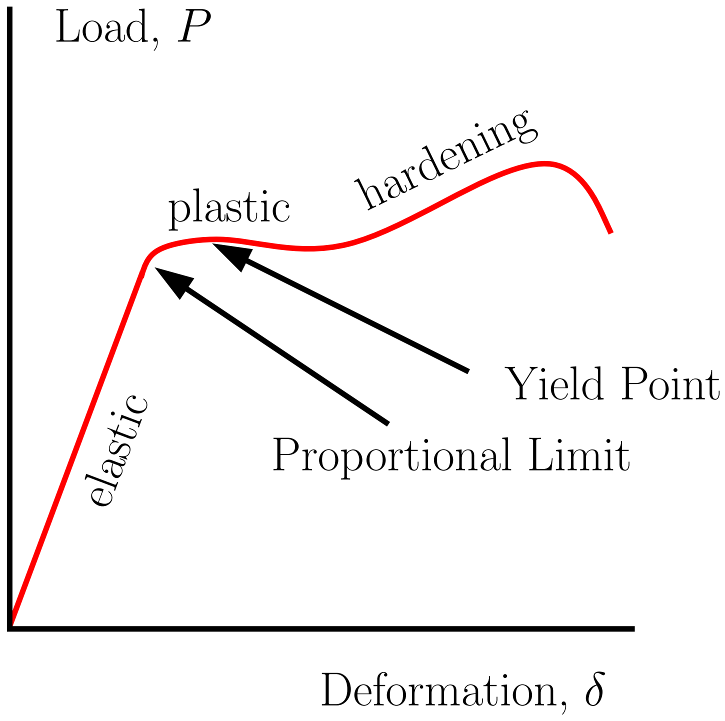

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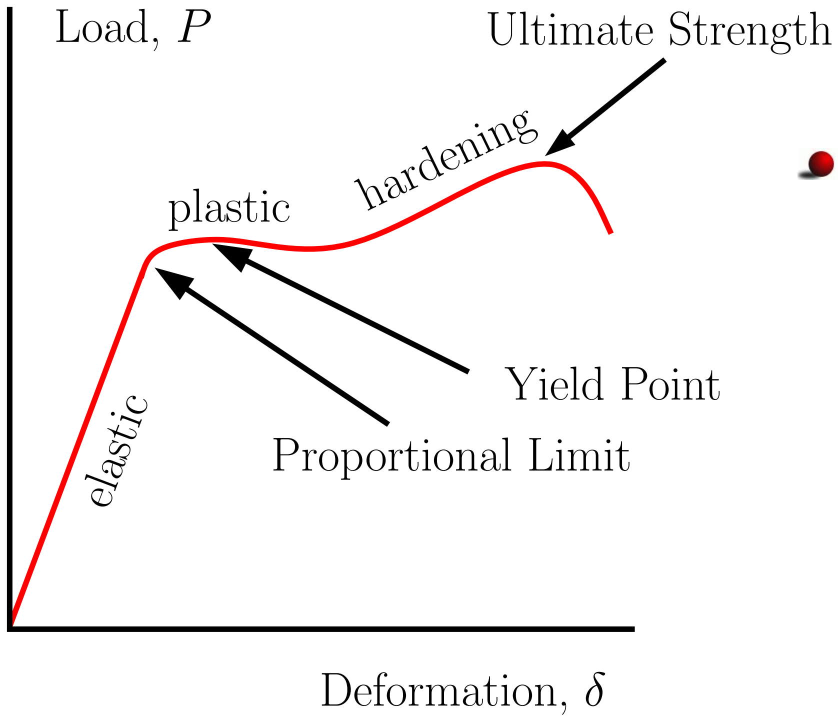

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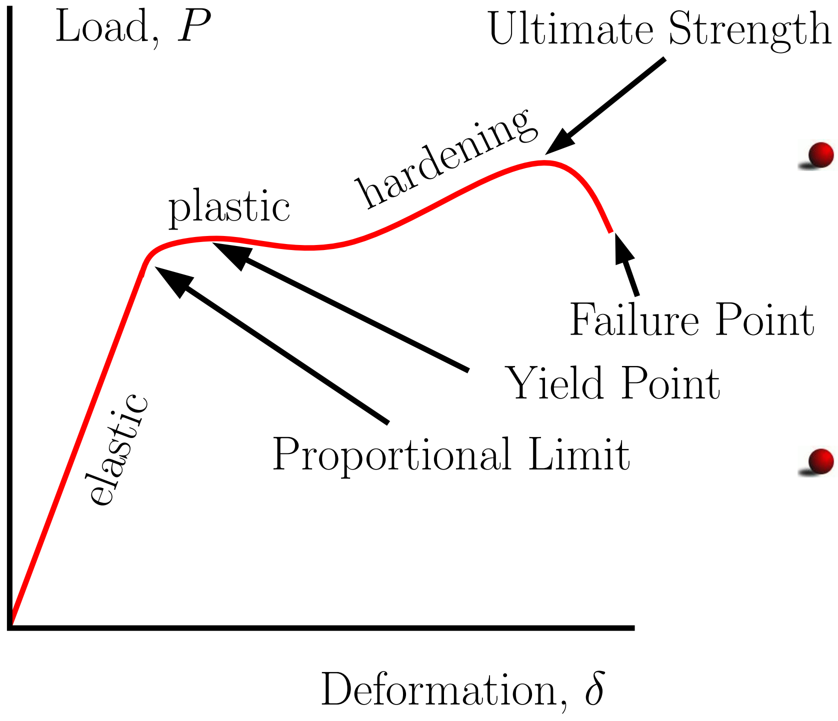
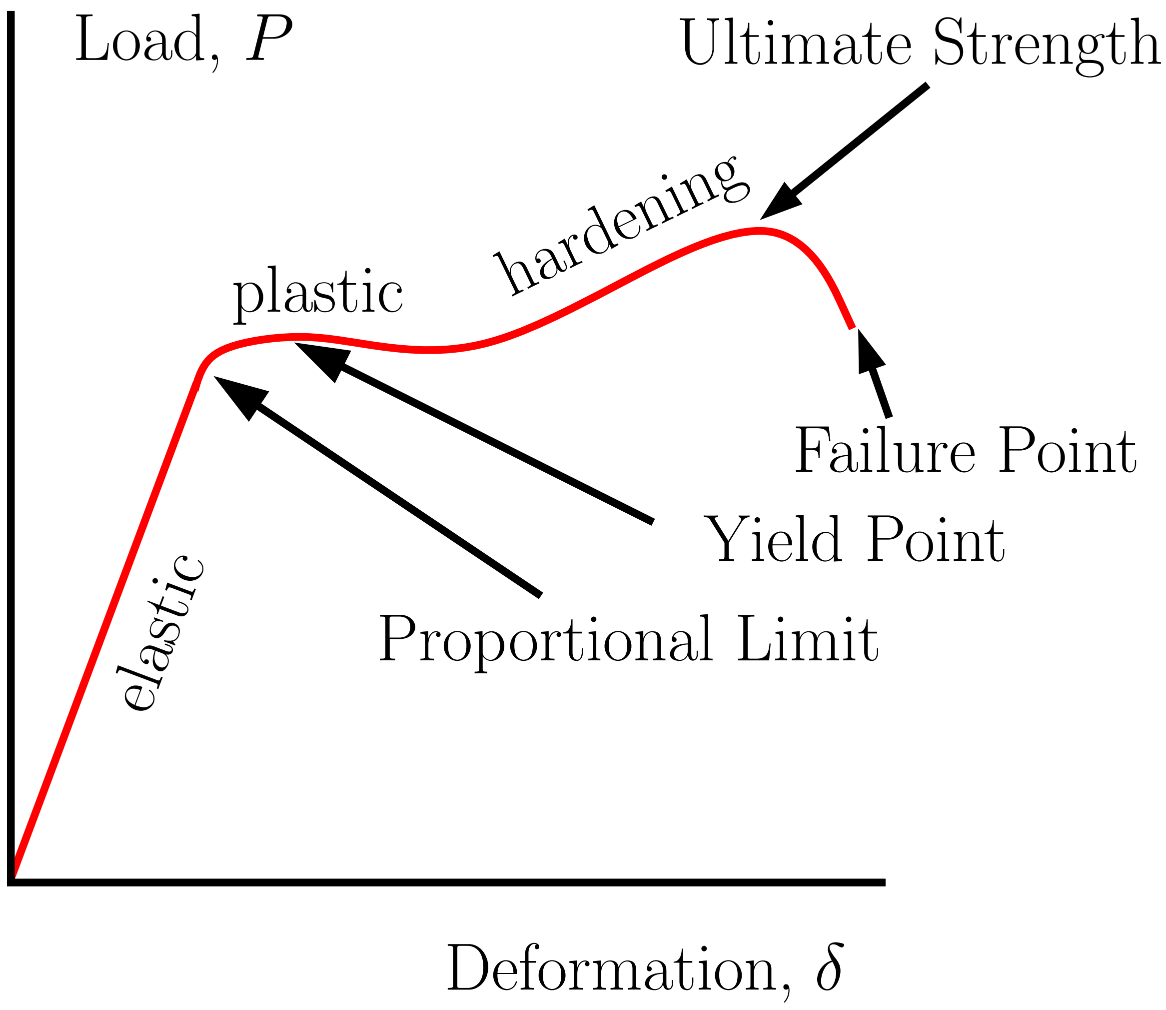
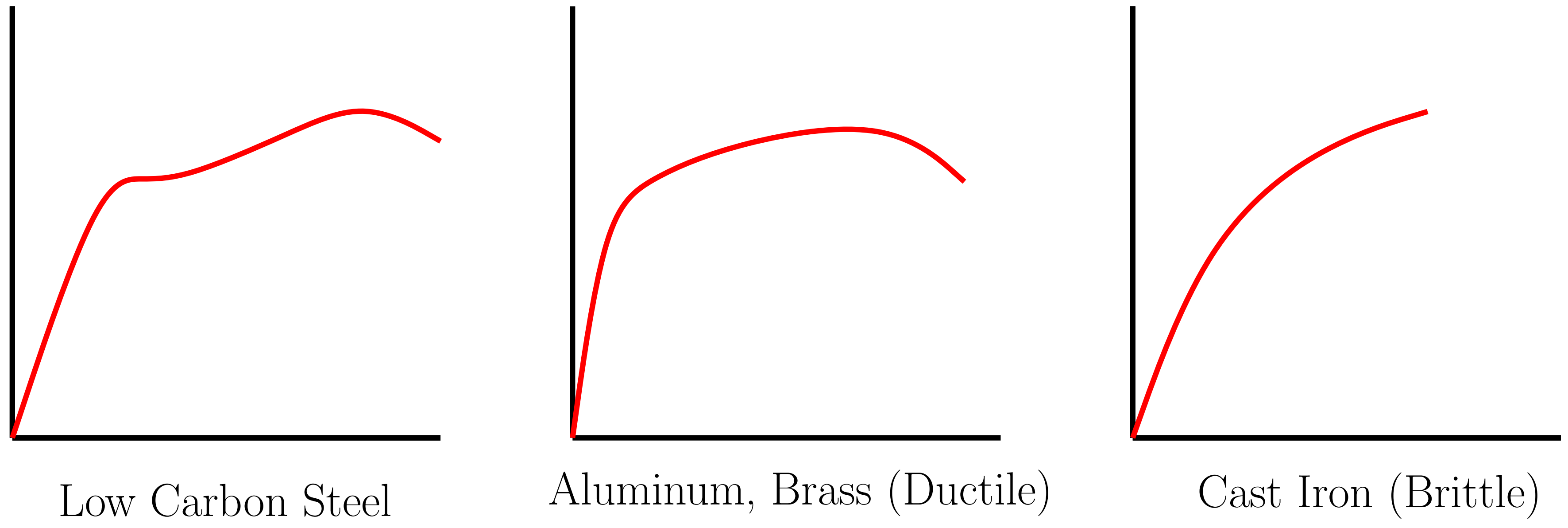


- This is the *strain-hardening* region
- Load (stress) rises to a maximum; this is the *ultimate strength* of the material
- Load required for further deformation is reduced as the *failure* or *breaking point* is approached.


Load, P
Ultimate Strength
hardening
plastic
Failure Point
Yield Point
elastic
Proportional Limit
Deformation, δ

Each material has its own stress-strain curve, with different characteristics:



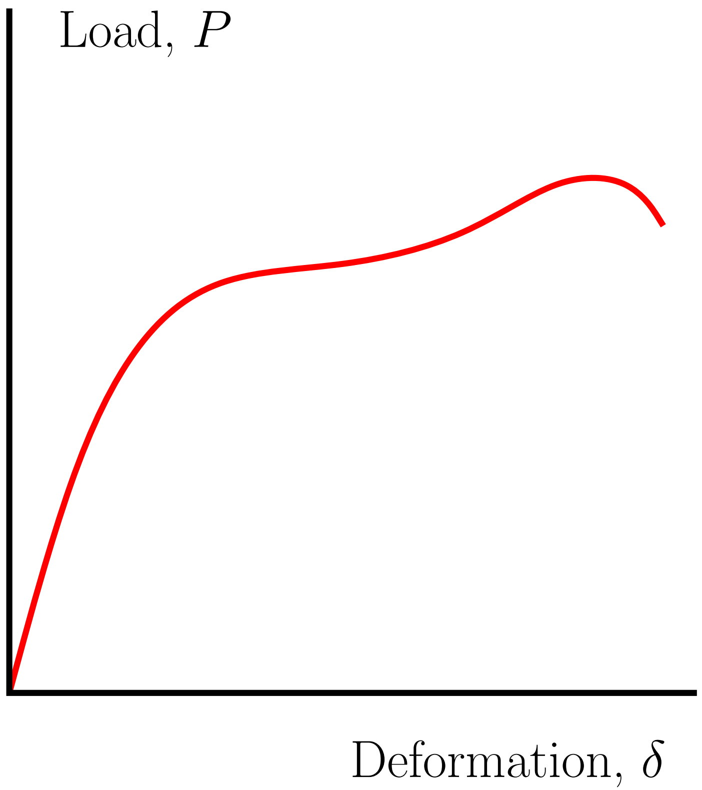
Ductile Materials:

- Can sustain large deformation before failure
- Deformation is an indication of impending failure
- Commonly used in construction
- Some steels, wood

Brittle Materials:

- Small deformation before failure
- Little indication of impending failure
- Cast iron, non-reinforced concrete

It is not always easy to identify the yield point from the stress-strain (load-deformation) curve. In these cases the ***offset method*** is used.



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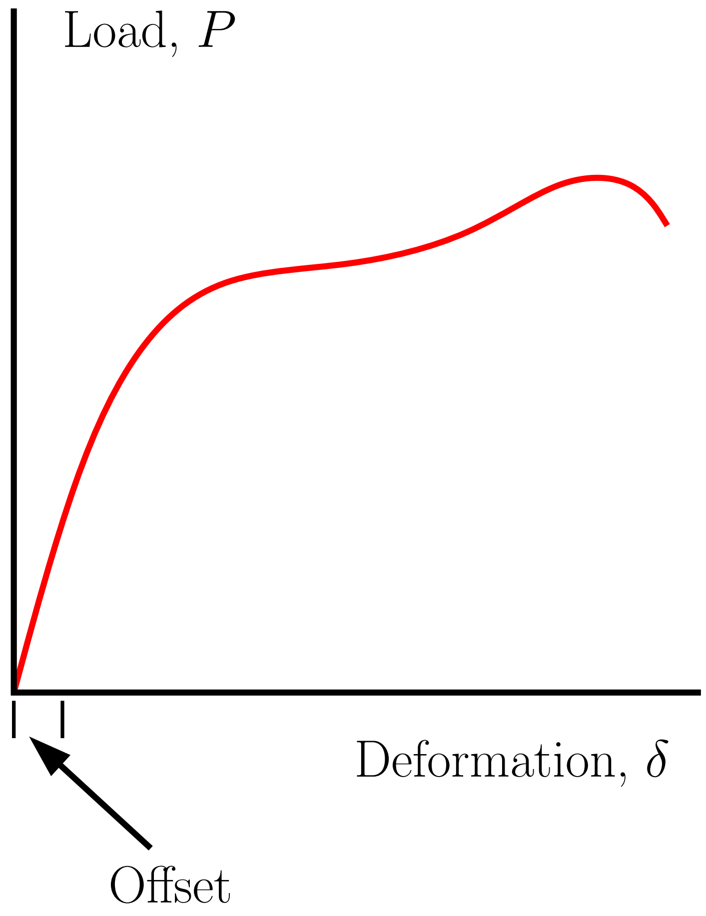

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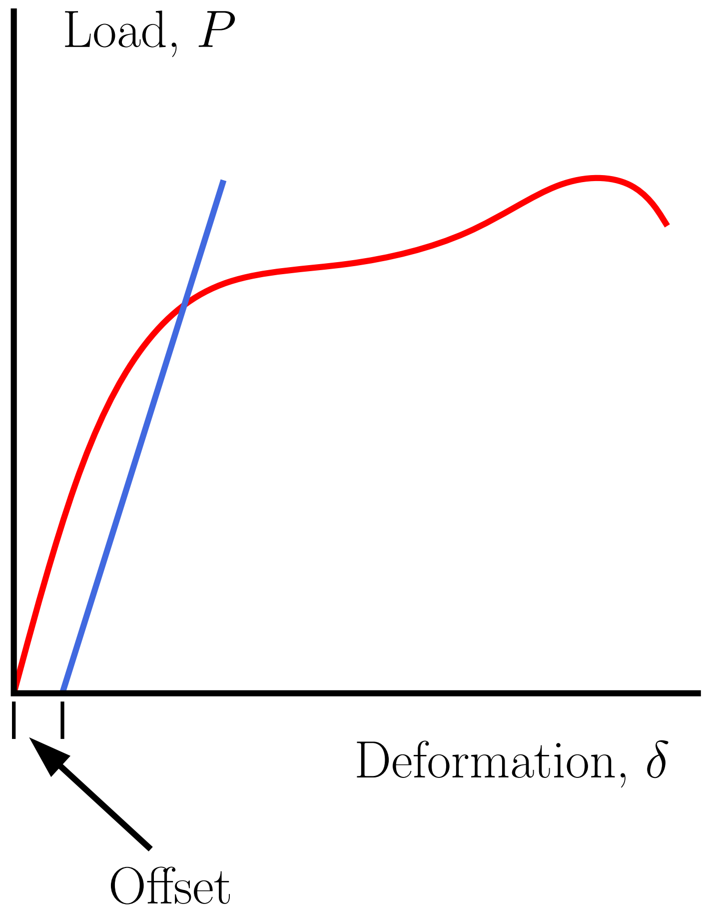

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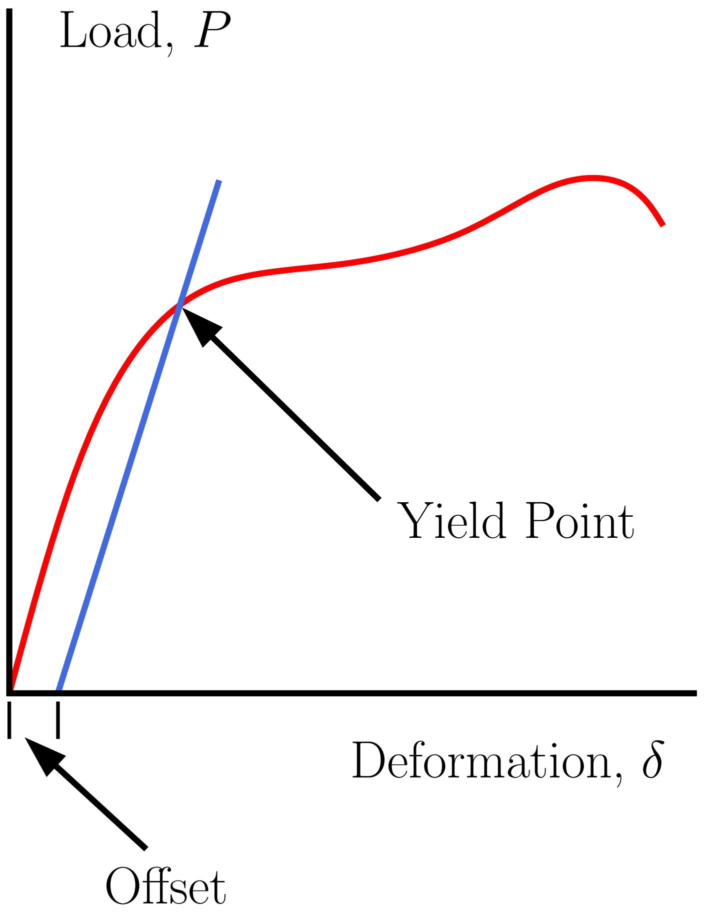
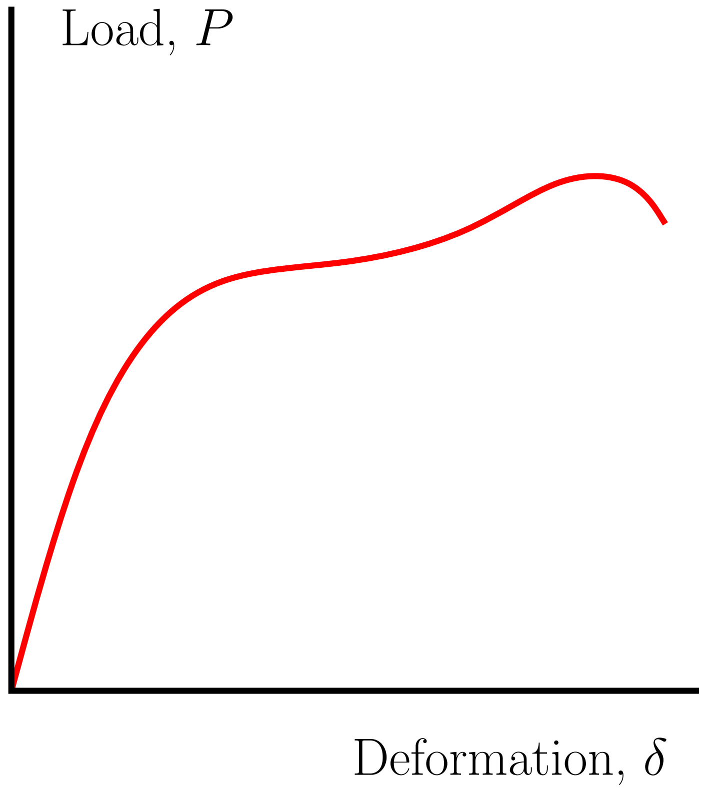


- An offset for the material is given. It is marked on the deformation (strain) axis
- A line through the offset point, parallel to the straight (proportional) part of the curve, is drawn
- The intersection of the line with the stress-strain curve is taken to be the yield point

For the tension and compression labs, we use the following offsets:



- For steel, use 0.2% strain
- For brass, use 0.35% strain
- For cast iron, use 0.05% strain

Tension and Compression Labs #1

You shall be provided with graphs showing the tension and compression curves from the lab

1. Draw horizontal and vertical axes
2. Mark each 10mm division
3. Each division along the horizontal axis represents a deformation of 0.04mm. Mark the divisions 0, 0.04, 0.08, etc.
   Note that the scale is $250 : 1$ (that is, $10 \; : \; 0.04$)
4. Load is on the vertical axis, with 10 in $= 120$ kN

$$(120/10) \times (1/25.4) = 0.472 \text{ kN/mm}$$

   Each division along the vertical axis represents a change in load of 4.72 kN. Mark the divisions 0, 4.72, 9.44, etc.

Tension and Compression Labs #2

Compute the location of the proportional limit:

1. Draw a vertical line through $\delta = 0.08$ (20mm division). The proportional limit is where this line crosses the curve. (This does not follow our theoretical description but is a common practice to find PL)
2. Calculate the strain at the PL
3. Calculate PL in MPa
4. Calculate E, the modulus of elasticity, in MPa

Tension and Compression Labs #3

1. Determine the yield point using the appropriate offset
2. Determine the deformation at the yield point
3. Calculate the strain at the yield point ($\epsilon = \delta / L_{original}$)
4. Calculate the ultimate strength
5. Calculate the stress at failure
6. Calculate the % elongation at failure
7. Calculate the % reduction in area at failure

Created by Dave Morgan using $\LaTeX\,2_\varepsilon$ and *Prosper* on February 24, 2005